

## UNIT-1(CHAPTERS 1 & 2)

Units And Measurements (Physics)

Question 2.1:

Fill in the blanks

The volume of a cube of side 1 cm is equal to.....m<sup>3</sup>

The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ... (mm)<sup>2</sup>

A vehicle moving with a speed of 18 km h<sup>-1</sup> covers....m in 1 s

The relative density of lead is 11.3. Its density is ....g cm<sup>-3</sup> or ...kg m<sup>-3</sup>.

**Answer**

$$1 \text{ cm} = \frac{1}{100} \text{ m}$$

Volume of the cube = 1 cm<sup>3</sup>

$$\text{But, } 1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = \left(\frac{1}{100}\right) \text{ m} \times \left(\frac{1}{100}\right) \text{ m} \times \left(\frac{1}{100}\right) \text{ m}$$

$$\therefore 1 \text{ cm}^3 = 10^{-6} \text{ m}^3$$

Hence, the volume of a cube of side 1 cm is equal to 10<sup>-6</sup> m<sup>3</sup>.

The total surface area of a cylinder of radius  $r$  and height  $h$  is

$$S = 2\pi r (r + h).$$

Given that,

$$r = 2 \text{ cm} = 2 \times 1 \text{ cm} = 2 \times 10 \text{ mm} = 20 \text{ mm}$$

$$h = 10 \text{ cm} = 10 \times 10 \text{ mm} = 100 \text{ mm}$$

$$\therefore S = 2 \times 3.14 \times 20 \times (20 + 100) = 15072 = 1.5 \times 10^4 \text{ mm}^2$$

Using the conversion,

$$1 \text{ km/h} = \frac{5}{18} \text{ m/s}$$

$$18 \text{ km/h} = 18 \times \frac{5}{18} = 5 \text{ m/s}$$

Therefore, distance can be obtained using the relation:

$$\text{Distance} = \text{Speed} \times \text{Time} = 5 \times 1 = 5 \text{ m}$$

Hence, the vehicle covers 5 m in 1 s.

Relative density of a substance is given by the relation,

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water}}$$

$$\text{Density of water} = 1 \text{ g/cm}^3$$

$$\begin{aligned} \text{Density of lead} &= \text{Relative density of lead} \times \text{Density of water} \\ &= 11.3 \times 1 = 11.3 \text{ g/cm}^3 \end{aligned}$$

$$\text{Again, } 1 \text{ g} = \frac{1}{1000} \text{ kg}$$

$$1 \text{ cm}^3 = 10^{-6} \text{ m}^3$$

$$1 \text{ g/cm}^3 = \frac{10^{-3}}{10^{-6}} \text{ kg/m}^3 = 10^3 \text{ kg/m}^3$$

$$\therefore 11.3 \text{ g/cm}^3 = 11.3 \times 10^3 \text{ kg/m}^3$$



Question 2.2:

Fill in the blanks by suitable conversion of units:

$$1 \text{ kg m}^2 \text{ s}^{-2} = \dots \text{ g cm}^2 \text{ s}^{-2}$$

$$1 \text{ m} = \dots \text{ ly}$$

$$3.0 \text{ m s}^{-2} = \dots \text{ km h}^{-2}$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 (\text{kg})^{-2} = \dots (\text{cm})^3 \text{ s}^{-2} \text{ g}^{-1}.$$

**Answer**

$$1 \text{ kg} = 10^3 \text{ g}$$

$$1 \text{ m}^2 = 10^4 \text{ cm}^2$$

$$1 \text{ kg m}^2 \text{ s}^{-2} = 1 \text{ kg} \times 1 \text{ m}^2 \times 1 \text{ s}^{-2}$$

$$= 10^3 \text{ g} \times 10^4 \text{ cm}^2 \times 1 \text{ s}^{-2} = 10^7 \text{ g cm}^2 \text{ s}^{-2}$$

Light year is the total distance travelled by light in one year.

$$1 \text{ ly} = \text{Speed of light} \times \text{One year}$$

$$= (3 \times 10^8 \text{ m/s}) \times (365 \times 24 \times 60 \times 60 \text{ s})$$

$$= 9.46 \times 10^{15} \text{ m}$$

$$\therefore 1 \text{ m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16} \text{ ly}$$

$$1 \text{ m} = 10^{-3} \text{ km}$$

$$\text{Again, } 1 \text{ s} = \frac{1}{3600} \text{ h}$$

$$1 \text{ s}^{-1} = 3600 \text{ h}^{-1}$$

$$1 \text{ s}^{-2} = (3600)^2 \text{ h}^{-2}$$

$$\therefore 3 \text{ m s}^{-2} = (3 \times 10^{-3} \text{ km}) \times ((3600)^2 \text{ h}^{-2}) = 3.88 \times 10^{-4} \text{ km h}^{-2}$$

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

$$1 \text{ kg} = 10^3 \text{ g}^{-1}$$

$$1 \text{ m}^3 = 10^6 \text{ cm}^3$$

$$\therefore 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} = 6.67 \times 10^{-11} \times (1 \text{ kg m s}^{-2}) (1 \text{ m}^2) (1 \text{ s}^{-2})$$

$$\begin{aligned}
&= 6.67 \times 10^{-11} \times (1 \text{ kg} \times 1 \text{ m}^3 \times 1 \text{ s}^{-2}) \\
&= 6.67 \times 10^{-11} \times (10^{-3} \text{ g}^{-1}) \times (10^6 \text{ cm}^3) \times (1 \text{ s}^{-2}) \\
&= 6.67 \times 10^{-8} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1}
\end{aligned}$$



Question 2.3:

A calorie is a unit of heat or energy and it equals about 4.2 J where  $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$ . Suppose we employ a system of units in which the unit of mass equals  $\alpha \text{ kg}$ , the unit of length equals  $\beta \text{ m}$ , the unit of time is  $\gamma \text{ s}$ . Show that a calorie has a magnitude  $4.2 \alpha^{-1} \beta^{-2} \gamma^2$  in terms of the new units.

**Answer**

Given that,

$$1 \text{ calorie} = 4.2 (1 \text{ kg}) (1 \text{ m}^2) (1 \text{ s}^{-2})$$

New unit of mass =  $\alpha \text{ kg}$

$$\text{Hence, in terms of the new unit, } 1 \text{ kg} = \frac{1}{\alpha} = \alpha^{-1}$$

In terms of the new unit of length,

$$1 \text{ m} = \frac{1}{\beta} = \beta^{-1} \text{ or } 1 \text{ m}^2 = \beta^{-2}$$

And, in terms of the new unit of time,

$$1 \text{ s} = \frac{1}{\gamma} = \gamma^{-1}$$

$$1 \text{ s}^2 = \gamma^{-2}$$

$$1 \text{ s}^{-2} = \gamma^2$$

$$\therefore 1 \text{ calorie} = 4.2 (1 \alpha^{-1}) (1 \beta^{-2}) (1 \gamma^2) = 4.2 \alpha^{-1} \beta^{-2} \gamma^2$$





Question 2.4:

Explain this statement clearly:

“To call a dimensional quantity ‘large’ or ‘small’ is meaningless without specifying a standard for comparison”. In view of this, reframe the following statements wherever necessary:

atoms are very small objects

a jet plane moves with great speed

the mass of Jupiter is very large

the air inside this room contains a large number of molecules

a proton is much more massive than an electron

the speed of sound is much smaller than the speed of light.

**Answer**

The given statement is true because a dimensionless quantity may be large or small in comparison to some standard reference. For example, the coefficient of friction is dimensionless. The coefficient of sliding friction is greater than the coefficient of rolling friction, but less than static friction.

An atom is a very small object in comparison to a soccer ball.

A jet plane moves with a speed greater than that of a bicycle.

Mass of Jupiter is very large as compared to the mass of a cricket ball.

The air inside this room contains a large number of molecules as compared to that present in a geometry box.

A proton is more massive than an electron.

Speed of sound is less than the speed of light.



Question 2.5:

A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

**Answer**

Distance between the Sun and the Earth:

= Speed of light  $\times$  Time taken by light to cover the distance

Given that in the new unit, speed of light = 1 unit

Time taken,  $t = 8 \text{ min } 20 \text{ s} = 500 \text{ s}$

$\therefore$  Distance between the Sun and the Earth =  $1 \times 500 = 500$  units



Question 2.6:

Which of the following is the most precise device for measuring length:

a vernier callipers with 20 divisions on the sliding scale

a screw gauge of pitch 1 mm and 100 divisions on the circular scale

an optical instrument that can measure length to within a wavelength of light ?

**Answer**

A device with minimum count is the most suitable to measure length.

Least count of vernier callipers

= 1 standard division (SD) – 1 vernier division (VD)

$$= 1 - \frac{9}{10} = \frac{1}{10} = 0.01 \text{ cm}$$

$$\text{Least count of screw gauge} = \frac{\text{Pitch}}{\text{Number of divisions}}$$

$$= \frac{1}{1000} = 0.001 \text{ cm}$$

$$\text{Least count of an optical device} = \text{Wavelength of light} \sim 10^{-5} \text{ cm}$$

$$= 0.00001 \text{ cm}$$

Hence, it can be inferred that an optical instrument is the most suitable device to measure length.



Question 2.7:

A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair?

**Answer**

Magnification of the microscope = 100

Average width of the hair in the field of view of the microscope = 3.5 mm

$$\therefore \text{Actual thickness of the hair is } \frac{3.5}{100} = 0.035 \text{ mm.}$$



Question 2.8:

Answer the following:

You are given a thread and a metre scale. How will you estimate the diameter of the thread?

A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing

the number of divisions on the circular scale?

The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

**Answer**

Wrap the thread on a uniform smooth rod in such a way that the coils thus formed are very close to each other. Measure the length of the thread using a metre scale. The diameter of the thread is given by the relation,

$$\text{Diameter} = \frac{\text{Length of thread}}{\text{Number of turns}}$$

It is not possible to increase the accuracy of a screw gauge by increasing the number of divisions of the circular scale. Increasing the number divisions of the circular scale will increase its accuracy to a certain extent only.

A set of 100 measurements is more reliable than a set of 5 measurements because random errors involved in the former are very less as compared to the latter.



Question 2.9:

The photograph of a house occupies an area of  $1.75 \text{ cm}^2$  on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is  $1.55 \text{ m}^2$ . What is the linear magnification of the projector-screen arrangement?

**Answer**

$$\text{Area of the house on the slide} = 1.75 \text{ cm}^2$$

$$\text{Area of the image of the house formed on the screen} = 1.55 \text{ m}^2$$

$$= 1.55 \times 10^4 \text{ cm}^2$$

$$\text{Area magnification, } m_a = \frac{\text{Area of image}}{\text{Area of object}} = \frac{1.55}{1.75} \times 10^4$$

$$\therefore \text{Linear magnification, } m_l = \sqrt{m_a}$$

$$= \sqrt{\frac{1.55}{1.75} \times 10^4} = 94.11$$



Question 2.10:

State the number of significant figures in the following:

0.007 m<sup>2</sup>

2.64 × 10<sup>24</sup> kg

0.2370 g cm<sup>-3</sup>

6.320 J

6.032 N m<sup>-2</sup>

0.0006032 m<sup>2</sup>

**Answer**

**Answer: 1**

The given quantity is 0.007 m<sup>2</sup>.

If the number is less than one, then all zeros on the right of the decimal point (but left to the first non-zero) are insignificant. This means that here, two zeros after the decimal are not significant. Hence, only 7 is a significant figure in this quantity.

**Answer: 3**

The given quantity is 2.64 × 10<sup>24</sup> kg.

Here, the power of 10 is irrelevant for the determination of significant figures. Hence, all

digits i.e., 2, 6 and 4 are significant figures.

**Answer:** 4

The given quantity is  $0.2370 \text{ g cm}^{-3}$ .

For a number with decimals, the trailing zeroes are significant. Hence, besides digits 2, 3 and 7, 0 that appears after the decimal point is also a significant figure.

**Answer:** 4

The given quantity is 6.320 J.

For a number with decimals, the trailing zeroes are significant. Hence, all four digits appearing in the given quantity are significant figures.

**Answer:** 4

The given quantity is  $6.032 \text{ Nm}^{-2}$ .

All zeroes between two non-zero digits are always significant.

**Answer:** 4

The given quantity is  $0.0006032 \text{ m}^2$ .

If the number is less than one, then the zeroes on the right of the decimal point (but left to the first non-zero) are insignificant. Hence, all three zeroes appearing before 6 are not significant figures. All zeros between two non-zero digits are always significant. Hence, the remaining four digits are significant figures.



Question 2.11:

The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

**Answer**

Length of sheet,  $l = 4.234 \text{ m}$

Breadth of sheet,  $b = 1.005$  m

Thickness of sheet,  $h = 2.01$  cm = 0.0201 m

The given table lists the respective significant figures:

Quantity	Number	Significant Figure
$l$	4.234	4
$b$	1.005	4
$h$	2.01	3

Hence, area and volume both must have least significant figures i.e., 3.

Surface area of the sheet =  $2(l \times b + b \times h + h \times l)$

$$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)$$

$$= 2(4.25517 + 0.020201 + 0.08510)$$

$$= 2 \times 4.360$$

$$= 8.72 \text{ m}^2$$

Volume of the sheet =  $l \times b \times h$

$$= 4.234 \times 1.005 \times 0.0201$$

$$= 0.0855 \text{ m}^3$$

This number has only 3 significant figures i.e., 8, 5, and 5.



Question 2.12:

The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

**Answer**

Mass of grocer's box = 2.300 kg

Mass of gold piece **I** = 20.15g = 0.02015 kg

Mass of gold piece **II** = 20.17 g = 0.02017 kg

Total mass of the box = 2.3 + 0.02015 + 0.02017 = 2.34032 kg

In addition, the final result should retain as many decimal places as there are in the number with the least decimal places. Hence, the total mass of the box is 2.3 kg.

Difference in masses = 20.17 – 20.15 = 0.02 g

In subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.



Question 2.13:

A physical quantity  $P$  is related to four observables  $a$ ,  $b$ ,  $c$  and  $d$  as follows:

$$P = \frac{a^3 b^2}{(\sqrt{cd})}$$

The percentage errors of measurement in  $a$ ,  $b$ ,  $c$  and  $d$  are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity  $P$ ? If the value of  $P$  calculated using the above relation turns out to be 3.763, to what value should you round off the result?

**Answer**

$$P = \frac{a^3 b^2}{(\sqrt{cd})}$$

$$\frac{\Delta P}{P} = \frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{1}{2} \frac{\Delta c}{c} + \frac{\Delta d}{d}$$

$$\left( \frac{\Delta P}{P} \times 100 \right) \% = \left( 3 \times \frac{\Delta a}{a} \times 100 + 2 \times \frac{\Delta b}{b} \times 100 + \frac{1}{2} \times \frac{\Delta c}{c} \times 100 + \frac{\Delta d}{d} \times 100 \right) \%$$

$$= 3 \times 1 + 2 \times 3 + \frac{1}{2} \times 4 + 2$$

$$= 3 + 6 + 2 + 2 = 13 \%$$

Percentage error in  $P = 13 \%$

Value of  $P$  is given as 3.763.

By rounding off the given value to the first decimal place, we get  $P = 3.8$ .



Question 2.14:

A book with many printing errors contains four different formulas for the displacement  $y$  of a particle undergoing a certain periodic motion:

$$y = a \sin\left(\frac{2\pi t}{T}\right)$$

$$y = a \sin vt$$

$$y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$$

$$y = (a\sqrt{2}) \left( \sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T} \right)$$

( $a$  = maximum displacement of the particle,  $v$  = speed of the particle.  $T$  = time-period of motion). Rule out the wrong formulas on dimensional grounds.

**Answer**

**Answer:** Correct

$$y = a \sin \frac{2\pi t}{T}$$

Dimension of  $y = M^0 L^1 T^0$

Dimension of  $a = M^0 L^1 T^0$

Dimension of  $\sin \frac{2\pi t}{T} = M^0 L^0 T^0$

∴ Dimension of L.H.S = Dimension of R.H.S

Hence, the given formula is dimensionally correct.

**Answer:** Incorrect

$$y = a \sin vt$$

Dimension of  $y = M^0 L^1 T^0$

Dimension of  $a = M^0 L^1 T^0$

Dimension of  $vt = M^0 L^1 T^{-1} \times M^0 L^0 T^1 = M^0 L^1 T^0$

But the argument of the trigonometric function must be dimensionless, which is not so in the given case. Hence, the given formula is dimensionally incorrect.

**Answer:** Incorrect

$$y = \left(\frac{a}{T}\right) \sin\left(\frac{t}{a}\right)$$

Dimension of  $y = M^0 L^1 T^0$

Dimension of  $\frac{a}{T} = M^0 L^1 T^{-1}$

Dimension of  $\frac{t}{a} = M^0 L^{-1} T^1$

But the argument of the trigonometric function must be dimensionless, which is not so in the given case. Hence, the formula is dimensionally incorrect.

**Answer:** Correct

$$y = (a\sqrt{2}) \left( \sin 2\pi \frac{t}{T} + \cos 2\pi \frac{t}{T} \right)$$

Dimension of  $y = M^0 L^1 T^0$

Dimension of  $a = M^0 L^1 T^0$

Dimension of  $\frac{t}{T} = M^0 L^0 T^0$

Since the argument of the trigonometric function must be dimensionless (which is true in the given case), the dimensions of  $y$  and  $a$  are the same. Hence, the given formula is dimensionally correct.



Question 2.15:

A famous relation in physics relates 'moving mass'  $m$  to the 'rest mass'  $m_0$  of a particle in terms of its speed  $v$  and the speed of light,  $c$ . (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant  $c$ . He writes:

$$m = \frac{m_0}{(1-v^2)^{\frac{1}{2}}}$$

**Answer**

Given the relation,

$$m = \frac{m_0}{(1-v^2)^{\frac{1}{2}}}$$

Dimension of  $m = M^1 L^0 T^0$

Dimension of  $m_0 = M^1 L^0 T^0$

Dimension of  $v = M^0 L^1 T^{-1}$

Dimension of  $v^2 = M^0 L^2 T^{-2}$

Dimension of  $c = M^0 L^1 T^{-1}$

The given formula will be dimensionally correct only when the dimension of L.H.S is the same as that of R.H.S. This is only possible when the factor,  $(1 - v^2/c^2)^{\frac{1}{2}}$  is dimensionless i.e.,  $(1 - v^2/c^2)$  is dimensionless. This is only possible if  $v^2$  is divided by  $c^2$ . Hence, the correct relation is

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$



Question 2.16:

The unit of length convenient on the atomic scale is known as an angstrom and is denoted by  $\text{\AA}$ :  $1 \text{\AA} = 10^{-10} \text{ m}$ . The size of a hydrogen atom is about  $0.5 \text{\AA}$  what is the total atomic volume in  $\text{m}^3$  of a mole of hydrogen atoms?

**Answer**

Radius of hydrogen atom,  $r = 0.5 \text{\AA} = 0.5 \times 10^{-10} \text{ m}$

Volume of hydrogen atom =  $\frac{4}{3} \pi r^3$

$$\begin{aligned} &= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3 \\ &= 0.524 \times 10^{-30} \text{ m}^3 \end{aligned}$$

1 mole of hydrogen contains  $6.023 \times 10^{23}$  hydrogen atoms.

$\therefore$  Volume of 1 mole of hydrogen atoms =  $6.023 \times 10^{23} \times 0.524 \times 10^{-30}$

$$= 3.16 \times 10^{-7} \text{ m}^3$$



Question 2.17:

One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen?

(Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large?

**Answer**

Radius of hydrogen atom,  $r = 0.5 \text{ Å} = 0.5 \times 10^{-10} \text{ m}$

Volume of hydrogen atom =  $\frac{4}{3}\pi r^3$

$$\begin{aligned} &= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3 \\ &= 0.524 \times 10^{-30} \text{ m}^3 \end{aligned}$$

Now, 1 mole of hydrogen contains  $6.023 \times 10^{23}$  hydrogen atoms.

$$\begin{aligned} \therefore \text{Volume of 1 mole of hydrogen atoms, } V_a &= 6.023 \times 10^{23} \times 0.524 \times 10^{-30} \\ &= 3.16 \times 10^{-7} \text{ m}^3 \end{aligned}$$

Molar volume of 1 mole of hydrogen atoms at STP,

$$V_m = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$$

$$\therefore \frac{V_m}{V_a} = \frac{22.4 \times 10^{-3}}{3.16 \times 10^{-7}} = 7.08 \times 10^4$$

Hence, the molar volume is  $7.08 \times 10^4$  times higher than the atomic volume. For this reason, the inter-atomic separation in hydrogen gas is much larger than the size of a hydrogen atom.

Question 2.18:

Explain this common observation clearly : If you look out of the window of a fast moving

train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

**Answer**

Line of sight is defined as an imaginary line joining an object and an observer's eye. When we observe nearby stationary objects such as trees, houses, etc. while sitting in a moving train, they appear to move rapidly in the opposite direction because the line of sight changes very rapidly.

On the other hand, distant objects such as trees, stars, etc. appear stationary because of the large distance. As a result, the line of sight does not change its direction rapidly.



Question 2.19:

The principle of 'parallax' in section 2.3.1 is used in the determination of distances of very distant stars. The baseline  $AB$  is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit  $\approx 3 \times 10^{11}$  m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1'' (second) of arc or so. A *parsec* is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1'' (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of meters?

**Answer**

Diameter of Earth's orbit =  $3 \times 10^{11}$  m

Radius of Earth's orbit,  $r = 1.5 \times 10^{11}$  m

Let the distance parallax angle be  $l'' = 4.847 \times 10^{-6}$  rad.

Let the distance of the star be  $D$ .

Parsec is defined as the distance at which the average radius of the Earth's orbit subtends

an angle of  $1''$ .

$$\therefore \text{We have } \theta = \frac{r}{D}$$

$$D = \frac{r}{\theta} = \frac{1.5 \times 10^{11}}{4.847 \times 10^{-6}} \\ = 0.309 \times 10^{-6} \approx 3.09 \times 10^{16} \text{ m}$$

Hence, 1 parsec  $\approx 3.09 \times 10^{16}$  m.



Question 2.20:

The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named *Alpha Centauri*) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

**Answer**

Distance of the star from the solar system = 4.29 ly

1 light year is the distance travelled by light in one year.

1 light year = Speed of light  $\times$  1 year

$$= 3 \times 10^8 \times 365 \times 24 \times 60 \times 60 = 94608 \times 10^{11} \text{ m}$$

$$\therefore 4.29 \text{ ly} = 405868.32 \times 10^{11} \text{ m}$$

$$\therefore 1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$$

$$\therefore 4.29 \text{ ly} = \frac{405868.32 \times 10^{11}}{3.08 \times 10^{16}} = 1.32 \text{ parsec}$$

Using the relation,

$$\theta = \frac{d}{D}$$

where,

Diameter of Earth's orbit,  $d = 3 \times 10^{11}$  m

Distance of the star from the Earth,  $D = 405868.32 \times 10^{11}$  m

$$\therefore \theta = \frac{3 \times 10^{11}}{405868.32 \times 10^{11}} = 7.39 \times 10^{-6} \text{ rad}$$

But, 1 sec =  $4.85 \times 10^{-6}$  rad

$$\therefore 7.39 \times 10^{-6} \text{ rad} = \frac{7.39 \times 10^{-6}}{4.85 \times 10^{-6}} = 1.52''$$



Question 2.21:

Precise measurements of physical quantities are a *need* of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

**Answer**

It is indeed very true that precise measurements of physical quantities are essential for the development of science. For example, ultra-short laser pulses (time interval  $\sim 10^{-15}$  s) are used to measure time intervals in several physical and chemical processes.

X-ray spectroscopy is used to determine the inter-atomic separation or inter-planer spacing.

The development of mass spectrometer makes it possible to measure the mass of atoms precisely.



Question 2.22:

Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations.

Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):

the total mass of rain-bearing clouds over India during the Monsoon

the mass of an elephant

the wind speed during a storm

the number of strands of hair on your head

the number of air molecules in your classroom.

### Answer

During monsoons, a metrologist records about 215 cm of rainfall in India i.e., the height of water column,  $h = 215 \text{ cm} = 2.15 \text{ m}$

Area of country,  $A = 3.3 \times 10^{12} \text{ m}^2$

Hence, volume of rain water,  $V = A \times h = 7.09 \times 10^{12} \text{ m}^3$

Density of water,  $\rho = 1 \times 10^3 \text{ kg m}^{-3}$

Hence, mass of rain water =  $\rho \times V = 7.09 \times 10^{15} \text{ kg}$

Hence, the total mass of rain-bearing clouds over India is approximately  $7.09 \times 10^{15} \text{ kg}$ .

Consider a ship of known base area floating in the sea. Measure its depth in sea (say  $d_1$ ).

Volume of water displaced by the ship,  $V_b = A d_1$

Now, move an elephant on the ship and measure the depth of the ship ( $d_2$ ) in this case.

Volume of water displaced by the ship with the elephant on board,  $V_{be} = A d_2$

Volume of water displaced by the elephant =  $A d_2 - A d_1$

Density of water =  $D$

Mass of elephant =  $AD (d_2 - d_1)$

Wind speed during a storm can be measured by an anemometer. As wind blows, it rotates. The rotation made by the anemometer in one second gives the value of wind speed.

Area of the head surface carrying hair =  $A$

With the help of a screw gauge, the diameter and hence, the radius of a hair can be determined. Let it be  $r$ .

∴ Area of one hair =  $\pi r^2$

Number of strands of hair  $\approx \frac{\text{Total surface area}}{\text{Area of one hair}} = \frac{A}{\pi r^2}$

Let the volume of the room be  $V$ .

One mole of air at NTP occupies 22.4 l i.e.,  $22.4 \times 10^{-3} \text{ m}^3$  volume.

Number of molecules in one mole =  $6.023 \times 10^{23}$

∴ Number of molecules in room of volume  $V$

$$= \frac{6.023 \times 10^{23}}{22.4 \times 10^{-3}} \times V = 134.915 \times 10^{26} V$$

$$= 1.35 \times 10^{28} V$$



Question 2.23:

The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding  $10^7$  K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun =  $2.0 \times 10^{30}$  kg, radius of the Sun =  $7.0 \times 10^8$  m.

**Answer**

Mass of the Sun,  $M = 2.0 \times 10^{30}$  kg

Radius of the Sun,  $R = 7.0 \times 10^8$  m

Volume of the Sun,  $V = \frac{4}{3} \pi R^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (7.0 \times 10^8)^3$$

$$= \frac{88}{21} \times 343 \times 10^{24} = 1437.3 \times 10^{24} \text{ m}^3$$

$$\text{Density of the Sun} = \frac{\text{Mass}}{\text{Volume}} = \frac{2.0 \times 10^{30}}{1437.3 \times 10^{24}} \sim 1.4 \times 10^3 \text{ kg/m}^3$$

The density of the Sun is in the density range of solids and liquids. This high density is attributed to the intense gravitational attraction of the inner layers on the outer layer of the Sun.



Question 2.24:

When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be  $35.72''$  of arc. Calculate the diameter of Jupiter.

**Answer**

Distance of Jupiter from the Earth,  $D = 824.7 \times 10^6 \text{ km} = 824.7 \times 10^9 \text{ m}$

Angular diameter =  $35.72'' = 35.72 \times 4.874 \times 10^{-6} \text{ rad}$

Diameter of Jupiter =  $d$

Using the relation,

$$\theta = \frac{d}{D}$$

$$d = \theta D = 824.7 \times 10^9 \times 35.72 \times 4.872 \times 10^{-6}$$

$$= 143520.76 \times 10^3 = 1.435 \times 10^5 \text{ km}$$



Question 2.25:

A man walking briskly in rain with speed  $v$  must slant his umbrella forward making an angle  $\theta$  with the vertical. A student derives the following relation between  $\theta$  and  $v$ :  $\tan \theta =$

$v$  and checks that the relation has a correct limit: as  $v \rightarrow 0$ ,  $\theta \rightarrow 0$ , as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

**Answer**

**Answer:** Incorrect; on dimensional ground

The relation is  $\tan \theta = v$ .

Dimension of R.H.S =  $M^0 L^1 T^{-1}$

Dimension of L.H.S =  $M^0 L^0 T^0$

( $\because$  The trigonometric function is considered to be a dimensionless quantity)

Dimension of R.H.S is not equal to the dimension of L.H.S. Hence, the given relation is not correct dimensionally.

To make the given relation correct, the R.H.S should also be dimensionless. One way to achieve this is by dividing the R.H.S by the speed of rainfall  $v'$ .

Therefore, the relation reduces to

$\tan \theta = \frac{v}{v'}$ . This relation is dimensionally correct.



Question 2.26:

It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?

**Answer**

Difference in time of caesium clocks = 0.02 s

Time required for this difference = 100 years

$$= 100 \times 365 \times 24 \times 60 \times 60 = 3.15 \times 10^9 \text{ s}$$

In  $3.15 \times 10^9$  s, the caesium clock shows a time difference of 0.02 s.

In 1s, the clock will show a time difference of  $\frac{0.02}{3.15 \times 10^9}$  s.

Hence, the accuracy of a standard caesium clock in measuring a time interval of 1 s is

$$\frac{3.15 \times 10^9}{0.02} = 157.5 \times 10^9 \text{ s} \approx 1.5 \times 10^{11} \text{ s}$$



Question 2.27:

Estimate the average mass density of a sodium atom assuming its size to be about  $2.5 \text{ \AA}$ . (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase:  $970 \text{ kg m}^{-3}$ . Are the two densities of the same order of magnitude? If so, why?

**Answer**

Diameter of sodium atom = Size of sodium atom =  $2.5 \text{ \AA}$

$$\text{Radius of sodium atom, } r = \frac{1}{2} \times 2.5 \text{ \AA} = 1.25 \text{ \AA}$$

$$= 1.25 \times 10^{-10} \text{ m}$$

$$\text{Volume of sodium atom, } V = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times 3.14 \times (1.25 \times 10^{-10})^3$$

According to the Avogadro hypothesis, one mole of sodium contains  $6.023 \times 10^{23}$  atoms and has a mass of 23 g or  $23 \times 10^{-3}$  kg.

$$\therefore \text{Mass of one atom} = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}} \text{ kg}$$

$$\text{Density of sodium atom, } \rho = \frac{\frac{23 \times 10^{-3}}{6.023 \times 10^{23}}}{\frac{4}{3} \times 3.14 \times (1.25 \times 10^{-10})^3} = 4.67 \times 10^{-3} \text{ kg m}^{-3}$$

It is given that the density of sodium in crystalline phase is  $970 \text{ kg m}^{-3}$ .

Hence, the density of sodium atom and the density of sodium in its crystalline phase are not in the same order. This is because in solid phase, atoms are closely packed. Thus, the inter-atomic separation is very small in the crystalline phase.



Question 2.28:

The unit of length convenient on the nuclear scale is a fermi :  $1 \text{ f} = 10^{-15} \text{ m}$ . Nuclear sizes

obey roughly the following empirical relation :  $r = r_0 A^{\frac{1}{3}}$

where  $r$  is the radius of the nucleus,  $A$  its mass number, and  $r_0$  is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise. 2.27.

**Answer**

Radius of nucleus  $r$  is given by the relation,

$$r = r_0 A^{\frac{1}{3}} \dots \text{(i)}$$

$$r_0 = 1.2 \text{ f} = 1.2 \times 10^{-15} \text{ m}$$

$$\text{Volume of nucleus, } V = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \pi \left( r_0 A^{\frac{1}{3}} \right)^3 = \frac{4}{3} \pi r_0^3 A \quad \dots \text{(i)}$$

Now, the mass of a nuclei  $M$  is equal to its mass number i.e.,

$$M = A \text{ amu} = A \times 1.66 \times 10^{-27} \text{ kg}$$

Density of nucleus,

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

$$= \frac{A \times 1.66 \times 10^{-27}}{\frac{4}{3} \pi r_0^3 A} = \frac{3 \times 1.66 \times 10^{-27}}{4 \pi r_0^3} \text{ kg/m}^3$$

This relation shows that nuclear mass depends only on constant  $r_0$ . Hence, the nuclear mass densities of all nuclei are nearly the same.

Density of sodium nucleus is given by,

$$\begin{aligned} \rho_{\text{Sodium}} &= \frac{3 \times 1.66 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3} \\ &= \frac{4.98}{21.71} \times 10^{18} = 2.29 \times 10^{17} \text{ kg m}^{-3} \end{aligned}$$



Question 2.29:

A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

**Answer**

Time taken by the laser beam to return to Earth after reflection from the Moon = 2.56 s

Speed of light =  $3 \times 10^8$  m/s

Time taken by the laser beam to reach Moon =  $\frac{1}{2} \times 2.56 = 1.28$  s

Radius of the lunar orbit = Distance between the Earth and the Moon =  $1.28 \times 3 \times 10^8 = 3.84 \times 10^8$  m =  $3.84 \times 10^5$  km



Question 2.30:

A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water =  $1450 \text{ m s}^{-1}$ ).

**Answer**

Let the distance between the ship and the enemy submarine be ' $S$ '.

Speed of sound in water = 1450 m/s

Time lag between transmission and reception of Sonar waves = 77 s

In this time lag, sound waves travel a distance which is twice the distance between the ship and the submarine ( $2S$ ).

Time taken for the sound to reach the submarine =  $\frac{1}{2} \times 77 = 38.5$  s

$\therefore$  Distance between the ship and the submarine ( $S$ ) =  $1450 \times 38.5 = 55825$  m = 55.8 km



Question 2.31:

The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach

us?

**Answer**

Time taken by quasar light to reach Earth = 3 billion years

$$= 3 \times 10^9 \text{ years}$$

$$= 3 \times 10^9 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$\text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

Distance between the Earth and quasar

$$= (3 \times 10^8) \times (3 \times 10^9 \times 365 \times 24 \times 60 \times 60)$$

$$= 283824 \times 10^{20} \text{ m}$$

$$= 2.8 \times 10^{22} \text{ km}$$

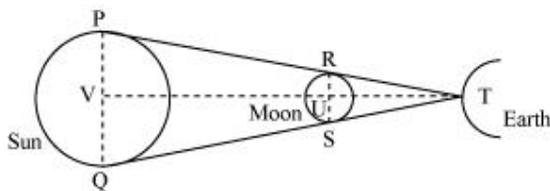


Question 2.32:

It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.

**Answer**

The position of the Sun, Moon, and Earth during a lunar eclipse is shown in the given figure.



Distance of the Moon from the Earth =  $3.84 \times 10^8$  m

Distance of the Sun from the Earth =  $1.496 \times 10^{11}$  m

Diameter of the Sun =  $1.39 \times 10^9$  m

It can be observed that  $\Delta TRS$  and  $\Delta TPQ$  are similar. Hence, it can be written as:

$$\frac{PQ}{RS} = \frac{VT}{UT}$$

$$\frac{1.39 \times 10^9}{RS} = \frac{1.496 \times 10^{11}}{3.84 \times 10^8}$$

$$RS = \frac{1.39 \times 3.84}{1.496} \times 10^6 = 3.57 \times 10^6 \text{ m}$$

Hence, the diameter of the Moon is  $3.57 \times 10^6$  m.



Question 2.33:

A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics ( $c$ ,  $e$ , mass of electron, mass of proton) and the gravitational constant  $G$ , he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe ( $\sim 15$  billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?

**Answer**

One relation consists of some fundamental constants that give the age of the Universe by:

$$t = \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \times \frac{1}{m_p m_e^2 c^3 G}$$

Where,

$t$  = Age of Universe

$e$  = Charge of electrons =  $1.6 \times 10^{-19}$  C

$\epsilon_0$  = Absolute permittivity

$m_p$  = Mass of protons =  $1.67 \times 10^{-27}$  kg

$m_e$  = Mass of electrons =  $9.1 \times 10^{-31}$  kg

$c$  = Speed of light =  $3 \times 10^8$  m/s

$G$  = Universal gravitational constant =  $6.67 \times 10^{-11}$  Nm<sup>2</sup> kg<sup>-2</sup>

Also,  $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$  Nm<sup>2</sup>/C<sup>2</sup>

Substituting these values in the equation, we get

$$\begin{aligned} t &= \frac{(1.6 \times 10^{-19})^4 \times (9 \times 10^9)^2}{(9.1 \times 10^{-31})^2 \times 1.67 \times 10^{-27} \times (3 \times 10^8)^3 \times 6.67 \times 10^{-11}} \\ &= \frac{(1.6)^4 \times 81}{9.1 \times 1.67 \times 27 \times 6.67} \times 10^{-76+18+62+27-24+11} \text{ s} \\ &= \frac{(1.6)^4 \times 81}{9.1 \times 1.67 \times 27 \times 6.67 \times 365 \times 24 \times 3600} \times 10^{-76+18+62+27-24+11} \text{ years} \\ &\approx 6 \times 10^{-9} \times 10^{18} \text{ years} \\ &= 6 \text{ billion years} \end{aligned}$$



## UNIT - I

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# UNIT AND MEASUREMENT

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- The dimensions of a physical quantity are the powers to which the fundamental (base) quantities are raised to represent that quantity.
- **Dimensionless Physical Quantities** : Angle, solid angle, relative density, specific gravity, strain, Poisson's ratio, Reynold's number, all trigonometric ratios, refractive index, mechanical efficiency, relative permittivity, dielectric constant, relative permeability, electric susceptibility, magnetic susceptibility.
- Physical quantity dimensionless need not to be unitless e.g. ANGLE but all unitless quantities are dimensionless.
- The **three main uses** of dimensional analysis are :

- (i) **Conversion of one system of units into another for which we use**

$$n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^a \left[ \frac{L_1}{L_2} \right]^b \left[ \frac{T_1}{T_2} \right]^c$$

where  $M_1, L_1, T_1$  are fundamental units on one system;  $M_2, L_2, T_2$  are fundamental units on the other system;  $a, b, c$  are the dimensions of the quantity in mass, length and time;  $n_1$  is numerical value of the quantity in one system and  $n_2$  is its numerical value in the other system.

- (ii) Checking the dimensional correctness of a given physical relation.
- (iii) Derivation of formulae.
- **Principle of Homogeneity of Dimensions** : According to this principle, a correct dimensional equation must be homogeneous, i.e., dimensions of all the terms in a physical expression must be same

Dim formula of LHS = Dim formula of RHS

- **Significant Figures** : In the measured value of a physical quantity, the digits about the correctness of which we are sure plus the last digit which

is doubtful, are called the significant figures. For counting significant figures rules are as :

1. All the non-zero digits are significant. In 2.738 the number of significant figures is 4.
2. All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all. As examples 209 and 3.002 have 3 and 4 significant figures respectively.
3. If the measurement number is less than 1, the zero(s) on the right of decimal point and to the left of the first non-zero digit are non-significant. In 0.00807, first three underlined zeros are non-significant and the number of significant figures is only 3.
4. The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 12.3 m = 1230 cm = 12300 mm has only 3 significant figures.
5. The trailing zero(s) in number with a decimal point are significant. Thus, 3.800 kg has 4 significant figures.
6. A choice of change of units does not change the number of significant digits or figures in a measurement.

#### **General Guidelines for using Symbols in S.I.?**

- (i) The unit names should not start with a capital letter, e.g., Newton, Joule, Coulomb etc., however their symbols can be written with capital letter e.g., N.J.C. etc.
- (ii) Symbols cannot be used in plural form, they are used in singular form only.
- (iii) No full stop should be put at the end of the symbol of the unit.
- (iv) No space should be given between a prefix and symbol of a unit e.g., km,  $\mu$ s, cm etc.
- (v) A prefix is not used alone. (any four)

#### **Objects which have masses in the range of $10^{-3}$ kg to $10^{-5}$ kg.**

$10^{-30}$  kg = an electron

- $10^{-27}$  kg = a proton  
 $10^{-27}$  kg = an atom  
 $10^{-13}$  kg = an RBC  
 $10^{-6}$  kg = a drop of rain  
 $10^{-5}$  kg = a mosquito  
 $10^{-3}$  kg = a grapes  
 $10^2$  kg = human being  
 $10^8$  kg = a large aircraft  
 $10^{23}$  kg = the moon  
 $10^{25}$  kg = the earth  
 $10^{30}$  kg = the sun  
 $10^{41}$  kg = our galaxy  
 $10^{55}$  kg = observable universe.

#### Limitations of Dimensional Analysis?

**Ans.**

- (i) If a physical quantity depends upon more than three different base quantities, the formula cannot be derived with the help of dimensional analysis.
- (ii) Dimensional analysis may not give the formula with true physical relationship, e.g., according to dimensional analysis,  $s = ut + \frac{1}{2}at^2$  is the correct relationship.
- (iii) Dimensionless constants like 1, 2, 3, ....., e,  $\pi$  are not available in the formula derived with the help of dimensional analysis.
- (iv) Formula containing trigonometric, exponential and 10 functions cannot be derived with the help of dimension analysis.
- (v) This method does not inform about the nature of the derived physical quantity, i.e., whether it is scalar or vector.

- In any mathematical operation involving addition, subtraction, decimal places in the result will correspond to lowest number of decimal places in any of the numbers involved.
- In a mathematical operation like multiplication and division, number of significant figures in the product or in the quotient will correspond to the smallest number of significant figures in any of the numbers involved.
- Problems with accuracy are due to errors. The precision describes the limitation of the measuring instrument.
- Difference between measured value and true value of a quantity represents error of measurement.

(i) Mean of n measurements

$$a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

$$= \frac{1}{n} \sum_{i=1}^n a_i$$

(ii) Mean absolute error

$$[|\Delta a_1| + |\Delta a_2| + |\Delta a_3|]$$

$$\Delta a_{\text{mean}} = \frac{[|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|]}{n}$$

$$= \frac{\sum_{i=1}^n |\Delta a_i|}{n}$$

(iii) Relative error =  $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$

(iv) Percentage error =  $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

- The errors are communicated in different mathematical operations as detailed below :

if  $\pm \Delta a$ ,  $\pm \Delta b$  and  $\pm \Delta x$  are absolute errors in  $a$ ,  $b$  and  $x$  respectively, then

(i) for  $x = (a + b)$ ,  $\Delta x = \pm [\Delta a + \Delta b]$

(ii) for  $x = (a - b)$ ,  $\Delta x = \pm [\Delta a + \Delta b]$

(iii) for  $x = a \times b$ ,  $\frac{\Delta x}{x} = \pm \left[ \frac{\Delta a}{a} + \frac{\Delta b}{b} \right]$

(iv) for  $x = a/b$ ,  $\frac{\Delta x}{x} = \pm \left[ \frac{\Delta a}{a} + \frac{\Delta b}{b} \right]$

(v) for  $x = \frac{a^n b^m}{c^p}$ ,  $\frac{\Delta x}{x} = \pm \left[ n \frac{\Delta a}{a} + m \frac{\Delta b}{b} + p \frac{\Delta c}{c} \right]$

### VERY SHORT ANSWER QUESTIONS (1 MARK)

1. Compare the relative strength of various fundamental forces.

<b>Ans.</b> Fundamental Forces		Relative Strength	
Strong nuclear forces	–	1	$\frac{1}{2}$
Electromagnetic forces	–	$10^{-2}$	$\frac{1}{2}$
Weak nuclear forces	–	$10^{-13}$	$\frac{1}{2}$
Gravitational forces	–	$10^{-38}$	$\frac{1}{2}$

2. If  $x = a + bt + ct^2$ , where  $x$  is in metre and in seconds, what is the unit of  $c$  ?

**Ans.** The unit of left hand side is metre so the units of  $ct^2$  should also be metre. Since  $t^2$  has unit of  $s^2$ , so the unit of  $c$  is  $m/s^2$ .

3. What is the difference between mN, Nm and nm ?

**Ans.** mN means milli newton,  $1 \text{ mN} = 10^{-3} \text{ N}$ , Nm means Newton meter nm means nano meter.

4. Give the order of magnitude of the following :

- (i) size of atom  
(ii) size of our galaxy.

5. How many kg make 1 unified atomic mass unit?
6. Name some physical quantities that have same dimension.
7. Name the physical quantities that have dimensional formula  $[ML^{-1} T^{-2}]$
8. Give two examples of dimension less variables.
9. State the number of significant figures in
  - (i)  $0.007m^2$
  - (ii)  $2.64 \times 10^{24} \text{ kg}$
  - (iii)  $0.2370 \text{ g cm}^{-3}$
  - (iv)  $0.2300m$
  - (v)  $86400$
  - (vi)  $86400 \text{ m}$
10. Given relative error in the measurement of length is .02, what is the percentage error?
11. A physical quantity P is related to four observables a, b, c and d as follows :

$$P = a^3 b^2 / (\sqrt{cd})$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P? What is the value of P calculated using the above relation turns out to be 3.763, to value should you round off the result ?

**Ans.** Relative error in P is given by

$$\frac{\Delta P}{P} = 3 \frac{\Delta a}{a} + 2 \frac{\Delta b}{b} + \frac{1}{2} \frac{\Delta c}{c} + \frac{\Delta d}{d}$$

So, percentage error

$$\begin{aligned} \frac{\Delta P}{P} \times 100 &= 3 \left( \frac{\Delta a}{a} \times 100 \right) + 2 \times \left( \frac{\Delta b}{b} \times 100 \right) + \frac{1}{2} \left( \frac{\Delta c}{c} \times 100 \right) + \frac{\Delta d}{d} \times 100 \\ &= (3 \times 1\%) + (2 \times 3\%) + \left( \frac{1}{2} \times 4\% \right) + (1 \times 2\%) \\ &= 13\% \end{aligned}$$

12. A boy recalls the relation for relativistic mass<sup>(m)</sup> in terms of rest mass ( $m_0$ ), velocity of particle v, but forgets to put the constant c (velocity of

light). He writes  $m = \frac{m_0}{(1 - v^2)^{1/2}}$  correct the equation by putting the missing 'c'.

13. Name the technique used in locating.
  - (a) an under water obstacle
  - (b) position of an aeroplane in space.
14. Deduce dimensional formulae of—
  - (i) Boltzmann's constant
  - (ii) mechanical equivalent of heat.
15. Give examples of dimensional constants and dimensionless constants.

### SHORT ANSWER QUESTIONS (2 MARKS)

1. What is a physical standard? What characteristics should it have?
2. Define the term unit. Distinguish between fundamental and derived units.
3. Describe the principle and use of SONAR and RADAR.
4. State the principle of homogeneity. Test the dimensional homogeneity of equations—

(i)  $S = ut + \frac{1}{2}at^2$

(ii)  $S_n = u + \frac{a}{2}(2n - 1)$

5. In van der Waals's gas equation  $\left(P + \frac{a}{v^2}\right)(v - b) = RT$ . Determine the dimensions of a and b.
6. Using dimensions convert (a) 1 newton into dynes (b) 1 erg into joules.
7. Magnitude of force experienced by an object moving with speed v is given by  $F = kv^2$ . Find dimensions of k.

8. A book with printing error contains four different formulae for displacement. Choose the correct formula/formulae

(a)  $y = a \sin \frac{2\pi}{T} t$                       (b)  $y = a \sin vt$

(c)  $y = \frac{a}{T} \sin\left(\frac{t}{a}\right)$                       (d)  $y = \frac{a}{T} \left( \sin \frac{2\pi}{T} t + \cos \frac{2\pi}{T} t \right)$

9. Give limitations of dimensional analysis.
10. For determination of 'g' using simple pendulum, measurements of length and time period are required. Error in the measurement of which quantity will have larger effect on the value of 'g' thus obtained. What is done to minimise this error?

### SHORT ANSWER QUESTIONS (3 MARKS)

1. Give the name of six Indian Scientists and their discoveries.
2. Name the discoveries made by the following scientists :
- (a) Faraday                      (b) Chadwick  
(c) Hubble                      (d) Maxwell  
(e) Newton                      (f) Bohr.
3. Name the scientific principle on which the following technology is based.
- (i) Steam engine                      (ii) Laser  
(iii) Aeroplane                      (iv) Rocket propulsion  
(v) Radio and T.V.                      (vi) Production of Ultra high magnetic field.
4. Describe a method for measuring the molecular size of Oleic acid.
5. What types of phenomena can be used as a time standard. What are the advantages of defining second in terms of period of radiation from cesium -133 atom.

6. Deduce the dimensional formula for the following quantities

- (i) Gravitational constant
- (ii) Yung's modules
- (iii) Coefficient of viscosity.

7. Define the following units :

- (i) Light year
- (ii) Parsec
- (iii) Astronomical unit (Au)

### LONG ANSWER QUESTIONS (5 MARKS)

1. Name the four basic forces in nature. Write a brief note of each. Hence compare their strengths and ranges.
2. Distinguish between the terms precision and accuracy of a measurement.
3. Explain
  - (i) absolute error
  - (ii) mean absolute error
  - (iii) relative error
  - (iv) percentage error
  - (v) random error

### NUMERICALS

1. Determine the number of light years in one metre.
2. The sides of a rectangle are  $(10.5 \pm 0.2)$  cm and  $(5.2 \pm 0.1)$  cm. Calculate its perimeter with error limits.
3. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces 20.15 g and 20.17 g are added to the box.
  - (i) What is the total mass of the box?
  - (ii) The difference in masses of the pieces to correct significant figures.
4. 5.74 g of a substance occupies  $1.2 \text{ cm}^3$ . Express its density to correct significant figures.
5. If displacement of a body  $s = (200 \pm 5)$  m and time taken by it  $t = (20 \pm 0.2)$  s, then find the percentage error in the calculation of velocity.

6. If the error in measurement of mass of a body be 3% and in the measurement of velocity be 2%. What will be maximum possible error in calculation of kinetic energy.
7. The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.49m, 2.50m and 2.48m. Find the average length, absolute error and percentage error. Express the result with error limit.
8. A physical quantity is measured as  $a = (2.1 \pm 0.5)$  units. Calculate the percentage error in (i)  $Q^2$  (2)  $2Q$ .
9. When the planet Jupiter is at a distance of 824.7 million km from the earth, its angular diameter is measured to be  $35.72''$  of arc. Calculate diameter of Jupiter.
10. A lesser light beamed at the moon takes 2.56 and to return after reflection at the moon's surface. What will be the radius of lunar orbit.
11. Convert
  - (i)  $3\text{m} \cdot \text{S}^{-2}$  to  $\text{km h}^{-2}$
  - (ii)  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$  to  $\text{cm}^3 \text{ g}^{-1} \text{ S}^{-2}$
12. A calorie is a unit of heat or energy and it equals 4.2 J where  $1\text{J} = 1\text{kg m}^2\text{S}^{-2}$ . Suppose we employ a system of units in which unit of mass is  $\alpha$  kg, unit of length is  $\beta$  m, unit of time is  $\gamma$  s. What will be magnitude of calorie in terms of this new system.
13. The escape velocity  $v$  of a body depends on–
  - (i) the acceleration due to gravity 'g' of the planet,
  - (ii) the radius R of the planet. Establish dimensionally the relation for the escape velocity.
14. The frequency of vibration of a string depends of on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.
15. One mole of an ideal gas at STP occupies 22.4 L. What is the ratio of molar volume to atomic volume of a mole of hydrogen? Why is the ratio so large. Take radius of hydrogen molecule to be  $1\text{\AA}$ .

### VERY SHORT ANSWER (1 MARK)

4. Size of atom =  $10^{-10}$  m (b) size of galaxy =  $10^{22}$  m  
Order = - 10, Order of magnitude = 22.
5.  $1u = 1.66 \times 10^{-27}$  kg
6. Work, energy and torque.
7. Stress, pressure, modulus of elasticity.
8. Strain, refractive index.
9. (i) 1, (ii) 3, (iii) 4, (iv) 4, (v) 3, (vi) 5 since it comes from a measurement the last two zeros become significant.
10. 2%
11. % error in measurement of  $X = a\alpha + b\beta + c\gamma$ .
12. Since quantities of similar nature can only be added or subtracted,  $v^2$  cannot be subtracted from 1 but  $v^2/c^2$  can be subtracted from 1.

$$\therefore m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

13. (a) SONAR → Sound Navigation and Ranging.  
(b) RADAR → Radio Detection and Ranging.
14. (i) Boltzmann Constant :

$$k = \frac{\text{Heat}}{\text{Temperature}} \Rightarrow [k] = \frac{ML^2T^{-2}}{K} = M^1L^2T^{-2}K^{-1}$$

$$(ii) [J] = \left[ \frac{\text{Work}}{\text{Heat}} \right] = \frac{M^1L^2T^{-2}}{M^1L^2T^{-2}} = [m^0L^0T^0]$$

15. Dimensional Constants : Gravitational constant, plank's constant.  
Dimensionless Constants :  $\pi$ , e.

### SHORT ANSWER (2 MARKS)

4. (i) Dimension of L.H.S. =  $[s] = [M^0L^1T^0]$   
 Dimension of R.H.S =  $[ut] + [at^2] = [M^0L^1T^{-1}] + [M^0L^1T^{-2} \cdot T^2]$   
 $= M^0L^1T^0$

$\therefore$  The equation to dimensionally homogeneous.

(ii)  $S_n =$  Distance travelled in  $n^{\text{th}}$  sec that is  $(S_n - S_{n-1})$

$\therefore S_n = u \times t + \frac{a}{2} (2n - 1) \times 1$

Hence this is dimensionally incorrect.

5. Since dimensionally similar quantities can only be added

$\therefore [P] = \left[ \frac{a}{v^2} \right] \Rightarrow [a] = [Pv^2] = M^1 L^5 T^{-2}$

$[b] = [v] = L^3$

7.  $[k] = \frac{[F]}{[v^2]} = \frac{M^1 L^1 T^{-2}}{[L T^{-1}]^2} = \frac{M^1 L^1 T^{-2}}{M^0 L^2 T^{-2}} = [M^1 L^{-1}]$

8. The argument of sine and cosine function must be dimension less so (a) is the probable correct formula. Since

(a)  $y = a \sin\left(\frac{2\pi}{T} t\right) \quad \therefore \left[\frac{2\pi t}{T}\right] = [T^0]$  is dimensionless.

(b)  $y = a \sin \mu t \quad \therefore [\mu t] = [L]$  is dimensional so this equation is incorrect.

(c)  $y = \frac{a}{t} \sin\left(\frac{t}{a}\right) \quad \left[\frac{t}{a}\right]$  is dimensional so this is incorrect.

(d)  $y = \frac{a}{t} \left( \sin \frac{2\pi}{T} t + \cos \frac{2\pi t}{T} \right)$  : Though  $\frac{2\pi t}{T}$  is dimensionless  $\frac{a}{T}$  does

not have dimensions of displacement so this is also incorrect

9. Limitation of dimensional analysis :-

1. The value of proportionality constant cannot be obtained
2. Equation containing sine and cosine, exponents, log etc. cannot be analysed.
3. It fails to derive the exact form of physical relation which depends on more than three fundamental quantities
4. It does not tell whether a quantity is scalar or vector.

### ANSWERS FOR NUMERICALS

#### (NUMERICAL)

1.  $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$

$$1 \text{ m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16} \text{ ly}$$

2.  $P = 2(l + b)$

$$= 2(10.5 + 5.2) + 2(0.2 + 0.1)$$

$$= (31.4 + 0.6) \text{ cm}$$

3. (i) Mass of box : 2.3 kg

$$\text{Mass of gold pieces} = 20.15 + 20.17 = 40.32 \text{ g} = 0.04032 \text{ kg.}$$

$$\text{Total mass} = 2.3 + 0.04032 = 2.34032 \text{ kg}$$

In correct significant figure mass = 2.3 kg (as least decimal)

(ii) Difference in mass of gold pieces = 0.02 g

In correct significant figure (2 significant fig. minimum decimal) will be 0.02 g.

4.  $\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{5.74}{1.2} = 4.783 \text{ g/cm}^3$

Here least significant figure is 2, so density = 4.8 g/cm<sup>3</sup>

5. Percentage error in measurement of displacement =  $\frac{5}{200} \times 100$

Percentage error in measurement of time =  $\frac{0.2}{20} \times 100$

$\therefore$  Maximum permissible error =  $2.5 + 1 = 3.5\%$

6. K.E. =  $\frac{1}{2}mv^2$

$\therefore \frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{2\Delta v}{v} \Rightarrow \frac{\Delta k}{k} \times 100 = \frac{\Delta m}{m} \times 100 + 2\left(\frac{\Delta v}{v}\right) \times 100$

$\therefore$  Percentage error in K.E. =  $3\% + 2 \times 2\% = 7\%$

7. Average length

$$= \frac{2.48 + 2.46 + 2.49 + 2.50 + 2.48}{5} = \frac{12.41}{5} = 2.48 \text{ m}$$

Mean absolute error

$$\frac{0.00 + 0.02 + 0.01 + 0.02 + 0.00}{5} = \frac{0.05}{5} = 0.013\text{n}$$

$$\begin{aligned} \text{Percentage error} &= \frac{0.01}{2.48} \times 100\% = 0.04 \times 100\% \\ &= 0.40\% \end{aligned}$$

Correct length =  $(2.48 \pm 0.01)\text{m}$

Correct length =  $(2.48 \text{ M} \pm 0.40\%)$

8.  $P = Q^2$

$$\frac{\Delta p}{p} = \frac{2\Delta Q}{Q} \quad \left(\frac{0.5}{2.1}\right) = \frac{1.0}{2.1} = 0.476$$

$$\frac{\Delta p}{p} \times 100\% = 47.6\% = 48\%$$

$R = 2Q$

$$\frac{\Delta R}{R} = \frac{\Delta Q}{Q} \Rightarrow \frac{0.5}{2.1} = 0.238$$

$$\frac{\Delta R}{R} \times 100\% = 24\%$$

9.  $Q = 35.72''$

$$1'' = 4.85 \times 10^{-6} \text{ radian} \Rightarrow = 35.72 \times 4.85 \times 10^{-6}$$

$$\begin{aligned} d = DQ &= 824.7 \times 10^6 \times 35.72 \times 4.85 \times 10^{-6} \\ &= 1.4287 \times 10^5 \text{ km} \end{aligned}$$

10.  $t = 2.56 \text{ s}$

$$\therefore t' = \text{time taken by laser beam to go to the man} = \frac{t}{2}$$

$$\begin{aligned} \text{distance between earth and moon} &= d = c \times \frac{t}{2} \\ &= 3 \times 10^8 \times \frac{2.56}{2} \\ &= 3.84 \times 10^8 \text{ m} \end{aligned}$$

$$\begin{aligned} 11. \quad (i) \quad 3 \text{ m s}^{-2} &= \left( \frac{3}{1000} \text{ km} \right) \left( \frac{1}{60 \times 60} \text{ hr} \right)^{-2} \\ &= \frac{3 \times (60 \times 60)^2}{1000} = 3.9 \times 10^4 \text{ km h}^{-2} \end{aligned}$$

$$\begin{aligned} (ii) \quad G &= 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \\ &= 6.67 \times 10^{-11} (\text{kg m s}^{-2})(\text{m}^2 \text{ kg}^{-2}) \\ &= 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \\ &= 6.67 \times 10^{-11} (1000 \text{ g})^{-1} (100 \text{ cm})^3 (\text{s}^{-2}) \\ &= 6.67 \times 10^{-11} \alpha \frac{1}{1000} \times 100 \times 100 \times 100 \end{aligned}$$

$$= 6.67 \times 10^{-8} \text{g}^{-1} \text{cm}^3 \text{s}^{-2}$$

$$12. \quad n_2 = n_1 \left[ \frac{m_1}{m_2} \right]^a \left( \frac{L_1}{L_2} \right)^b \left( \frac{T_1}{T_2} \right)^c$$

$$= 4.2 \left( \frac{\text{kg}}{\alpha \text{kg}} \right)^1 \left( \frac{\text{m}}{\beta \text{m}} \right)^2 \left( \frac{\text{s}}{\text{rs}} \right)^{-2}$$

$$n_2 = 4.2 \alpha^{-1} \beta^{-2} \rho^{+2}$$

$$13. \quad v \propto g^a R^b \quad \mu = k g^a R^b \quad K \rightarrow \text{dimensionless proportionality constant}$$

$$[v] = [g]^a [R]^b$$

$$[M^0 L^1 T^{-1}] = [M^0 L^1 T^{-2}]^a [m^0 L^1 T^0]^b$$

equating powers

$$1 = a + b$$

$$-1 = -2a \Rightarrow a = \frac{1}{2}$$

$$b = 1 - a = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore v = k\sqrt{gR}$$

$$14. \quad n \propto I^a T^b m^c [l] = M^0 L^1 T^0$$

$$[T] = M^1 L^1 T^{-2} \text{ (force)}$$

$$[M] = M^1 L^{-1} T^0$$

$$[M^0 L^1 T^{-1}] = [M^0 L^1 T^0]^a [M^1 L^1 T^{-2}]^b [M^1 L^{-1} T^0]^c$$

$$b + c = 0$$

$$a + b - c = 0$$

$$-2b = -1 \quad b = 1/2$$

$$c = -\frac{1}{2} \quad a = 1$$

$$n \propto \frac{1}{l} \sqrt{\frac{T}{M}}$$

15.  $1 \text{ \AA} = 10^{-10} \text{ m}$

Atomic volume of 1 mole of hydrogen = Avagadios number  $\times$  volume of hydrogen molecule

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times \pi \times (10^{-10})^3$$

$$= 25.2 \times 10^{-7} \text{ m}^3$$

$$\text{Molar volume} = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$$

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{-7}} = 0.89 \times 10^4 \approx 10^4$$

This ratio is large because actual size of gas molecule is negligible in comparison to the inter molecular separation.