19

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Electricity is the most convenient form of energy available to us. It lights our houses, runs trains, operates communication devices and makes our lives comfortable. The list of electrical appliances that we use in our homes is very long. Have you ever thought as to how is electricity produced?

Hydro-electricity is produced by a generator which is run by a turbine using the energy of water. In a coal, gas or nuclear fuel power station, the turbine uses steam to run the generator. Electricity reaches our homes through cables from the town substation. Have you ever visited an electric sub-station? What are the big machines installed there? These machines are called transformers. Generators and transformers are the devices, which basically make electricity easily available to us. These devices are based on the principle of electromagnetic induction.

In this lesson you will study electromagnetic induction, laws governing it and the devices based on it. You will also study the construction and working of electric generators, transformers and their role in providing electric power to us. A brief idea of eddy current and its application will also be undertaken in this chapter.

OBJECTIVES

After studying this lesson, you should be able to :

- *explain the phenomenon of electromagnetic induction with simple experiments;*
- *explain Faraday's and Lenz's laws of electromagnetic induction;*
- *explain eddy currents and its applicaitons;*

- *describe the phenomena of self-induction and mutual induction;*
- *describe the working of ac and dc generators;*
- *derive relationship between voltage and current in ac circuits containing a (i) resistor, (ii) inductor, and or (iii) capacitor;*
- *analyse series LCR circuits; and*
- *explain the working of transformers and ways to improve their efficiency.*

19.1 ELECTROMAGNETIC INDUCTION

In the previous lesson you have learnt that a steady current in a wire produces a steady magnetic field. Faraday initially (and mistakenly) thought that a steady magnetic field could produce electric current . Some of his investigations on magnetically induced currents used an arrangement similar to the one shown in Fig.19.1. A current in the coil on the left produces a magnetic field concentrated in the iron ring. The coil on the right is connected to a galvanometer *G*, which can indicate the presence of an induced current in that circuit. It is observed that there is no deflection in *G* for a steady current flow but when the switch S in the left circuit is closed, the galvanometer shows deflection for a moment. Similarly, when switch *S* is opened, momentary deflection is recorded but in opposite direction. It means that current is induced only when the magnetic field due to the current in the circuit on the left ***changes.***

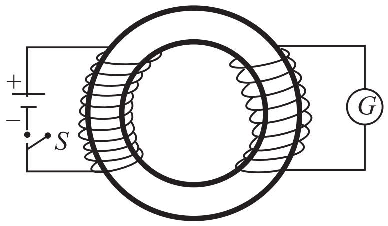


Fig. 19.1: Two coils are wrapped around an iron ring. The galvanometer *G* deflects for a moment when the switch is opened or closed.

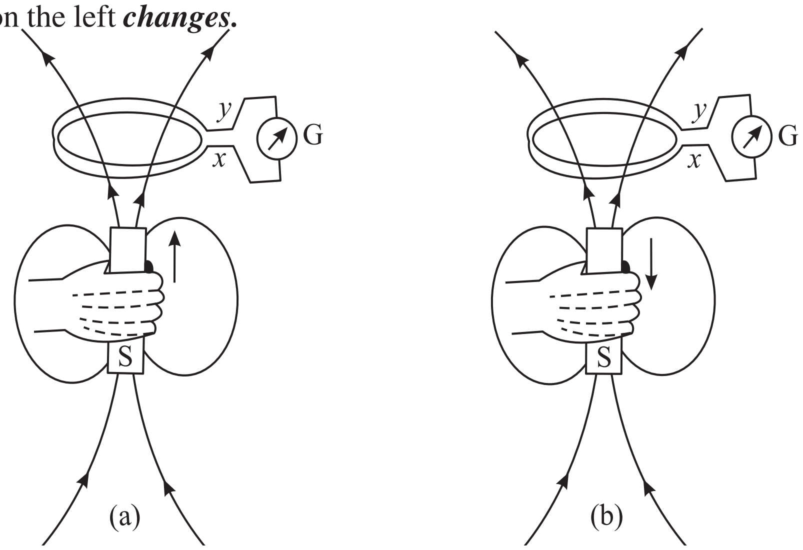
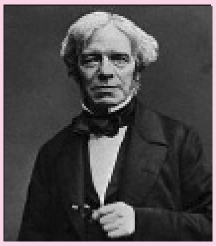


Fig. 19.2 : **a)** A current is induced in the coil if the magnet moves towards the coil, and **b)** the induced current has opposite direction if the magnet moves away from the coil.

The importance of a change can also be demonstrated by the arrangement shown in Fig.19.2. If the magnet is at rest relative to the coil, no current is induced in the coil. But when the magnet is moved towards the coil, current is induced in the direction indicated in Fig. 19.2a. Similarly, if the magnet is moved away from the coil, the a current is induced in the opposite direction, as shown in Fig.19.2b. Note that in both cases, the magnetic field changes in the neighbourhood of the coil. An induced current is also observed to flow through the coil, if this is moved relative to the magnet. The presence of such currents in a circuit implies the existence of an ***induced electromotive force (emf)*** across the free ends of the coil, i.e., x and y.

This phenomenon in which a magnetic field induces an emf is termed as ***electromagnetic induction***. Faraday's genius recognised the significance of this work and followed it up. The quantitative description of this phenomenon is known as Faraday's law of electromagnetic induction. We will discuss it now.



Michael Faraday (1791-1867)

British experimental scientist Michael Faraday is a classical example of a person who became great by shear hardwork, perseverance, love for science and humanity. He started his carrier as an apprentice with a book binder, but utilized the opportunity to read science books that he received for binding. He sent his notes to Sir Humphry Davy, who immediately recognised the talent in the young man and appointed him his permanent assistant in the Royal Institute.

Sir Humphry Davy once admitted that the greatest discovery of his life was Michael Faraday. And he was right because Faraday made basic discoveries which led to the electrical age. It is because of his discoveries that electrical generators, transformers, electrical motors, and electolysis became possible.

19.1.1 Faraday's Law of Electromagnetic Induction

The relationship between the changing magnetic field and the induced emf is expressed in terms of magnetic flux ϕ_B linked with the surface of the coil. You will now ask: What is magnetic flux? To define ***magnetic flux*** ϕ_B refer to Fig. 19.3a, which shows a typical infinitesimal element of area **ds**, into which the given surface can be considered to be divided. The direction of **ds** is normal to the surface at that point. By analogy with electrostatics, we can define the magnetic flux $d\phi_B$ for the area element **ds** as

$$d\phi_B = \mathbf{B} . \mathbf{ds} \tag{19.1a}$$

The magnetic flux for the entire surface is obtained by summing such contributions over the surface. Thus,

$$d\phi_B = \sum \mathbf{B}.\mathbf{ds} \qquad (19.1b)$$

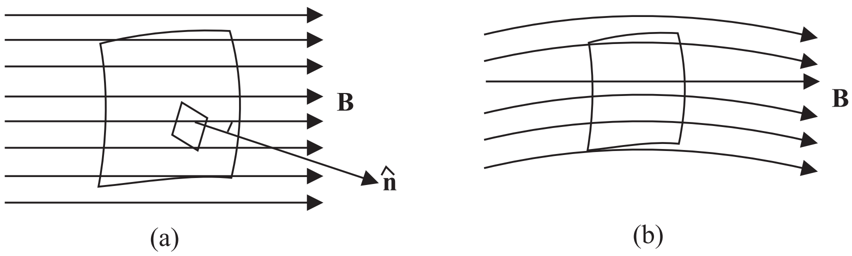


Fig. 19.3: a) The magnetic flux for an infinitesimal area ds is given by $d\phi_B$ = B.ds, and **b)** The magnetic flux for a surface is proportional to the number of lines intersecting the surface.

The SI unit of magnetic flux is **weber** (Wb), where 1 Wb = 1 Tm^2.

In analogy with electric lines and as shown in Fig.19.3b, the number of magnetic lines intersecting a surface is proportional to the magnetic flux through the surface.

Faraday's law states that *an emf is induced across a loop of wire when the magnetic flux linked with the surface bound by the loop changes with time. The magnitude of induced emf is proportional to the rate of change of magnetic flux.* Mathematically, we can write

$$|\varepsilon| = \frac{d\phi_B}{dt} \qquad (19.3)$$

From this we note that weber (Wb), the unit of magnetic flux and volt (V), the unit of emf are related as 1V = 1Wb s^{-1}.

Now consider that an emf is induced in a closely wound coil. Each turn in such a coil behaves approximately as a single loop, and we can apply Faraday's law to determine the emf induced in each turn. Since the turns are in series, the total induced emf ε_r in a coil will be equal to the sum of the emfs induced in each turn. We suppose that the coil is so closely wound that the magnetic flux linking each turn of the coil has the same value at a given instant. Then the same emf ε is induced in each turn, and the total induced emf for a coil with *N* turns is given by

$$|\varepsilon_r| = N|\varepsilon| = N\left(\frac{d\phi_B}{dt}\right) \qquad (19.4)$$

where ϕ_B is the magnetic flux linked with a single turn of the coil.

Let us now apply Faraday's law to some concrete situations.

Example 19.1 : The axis of a 75 turn circular coil of radius 35mm is parallel to a uniform magnetic field. The magnitude of the field changes at a constant rate from 25mT to 50 mT in 250 millisecond. Determine the magnitude of induced emf in the coil in this time interval.

Solution : Since the magnetic field is uniform and parallel to the axis of the coil, the flux linking each turn is given by

$$\phi_B = B\pi R^2$$

where R is radius of a turn. Using Eq. (19.4), we note that the induced emf in the coil is given by

$$|\varepsilon_r| = N\frac{d\phi_B}{dt} = N\frac{d(B\pi R^2)}{dt} = N\,\pi\,R^2\frac{dB}{dt} = N\,\pi\,R^2\left(\frac{B_2 - B_1}{t}\right)$$

Hence, the magnitude of the emf induced in the coil is

$$|\varepsilon_r| = 75\pi\,(0.035\text{m})^2\,(0.10\text{Ts}^{-1}) = 0.030\text{V} = 30\text{mV}$$

This example explains the concept of emf induced by a time changing magnetic field.

Example 19.2 : Consider a long solenoid with a cross-sectional area 8cm^2 (Fig. 19.4a and 19.4b). A time dependent current in its windings creates a magnetic field $B(t) = B_0 \sin 2\pi\nu t$. Here B_0 is constant, equal to 1.2 T. and ν, the frequency of the magnetic field, is 50 Hz. If the ring resistance $R = 1.0\Omega$, calculate the emf and the current induced in a ring of radius r concentric with the axis of the solenoid.

Solution : We are told that magnetic flux

$$\phi_B = B_0 \sin 2\pi\nu t A$$

since normal to the cross sectional area of the solenoid is in the direction of magnetic field.

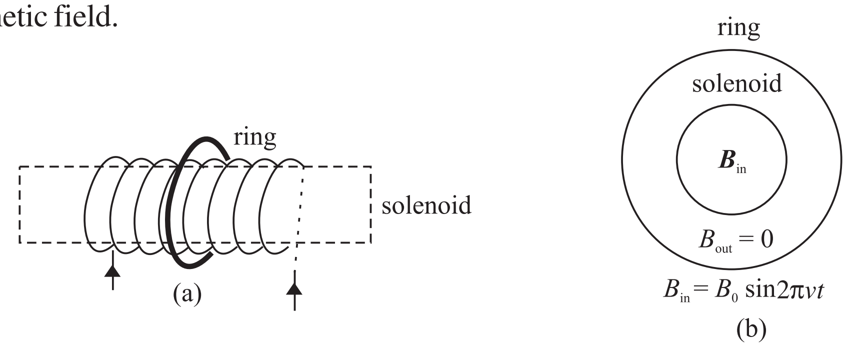


Fig.19.4 : **a)** A long solenoid and a concentric ring outside it, and **b)** cross-sectional view of the solenoid and concentric ring.

$$\text{Hence } |\varepsilon| = \frac{d\phi_B}{dt} = 2\pi\nu AB_0 \cos 2\pi\nu t.$$

$$= 2\pi\,.\,(50\text{s}^{-1})\,(8\times10^{-4}\text{m}^2)\,(1.2\text{ T})\cos 2\pi\nu t$$
$$= 0.3 \cos 2\pi\nu t \text{ volts}$$
$$= 0.3 \cos 100\pi t \text{ V}$$

The current in the ring is $I = \varepsilon/R$. Therefore

$$I = \frac{(0.3\cos 100\pi t)\ \text{V}}{(1.0\Omega)}$$

$$= +0.3 \cos 100\,\pi\, t\ \ \text{A}$$

INTEXT QUESTIONS 19.1

1. A 1000 turn coil has a radius of 5 cm. Calculate the emf developed across the coil if the magnetic field through the coil is reduced from 10 T to 0 in (a) 1s (b) 1ms.

2. The magnetic flux linking each loop of a 250-turn coil is given by $\phi_B(t) = A + Dt^2$, where A = 3 Wb and D = 15 Wbs^{-2} are constants. Show that a) the magnitude of the induced emf in the coil is given by $\varepsilon = (2ND)t$, and b) evaluate the emf induced in the coil at t = 0s and t = 3.0s.

3. The perpendicular to the plane of a conducting loop makes a fixed angle θ with a spatially uniform magnetic field. If the loop has area S and the magnitude of the field changes at a rate dB/dt, show that the magnitude of the induced emf in the loop is given by $\varepsilon = (dB/dt)\ S \cos\theta$. For what orientation(s) of the loop will ε be a) maximum and b) minimum?

19.1.2 Lenz's Law

Consider a bar magnet approaching a conducting ring (Fig.19.5a). To apply Faraday's law to this system, we first choose a positive direction with respect to the ring. Let us take the direction from O to Z as positive. (Any other choice is fine, as long as we are consistent.) For this configuration, the positive normal for the area of the ring is in the z-direction and the magnetic flux is negative. As the distance between the conducting ring and the N-pole of the bar magnet decreases, more and more field lines go though the ring, making the flux more and more negative. Thus $d\phi_B/dt$ is negative. By Faraday's law, ε is positive relative to our chosen direction. The current I is directed as shown.

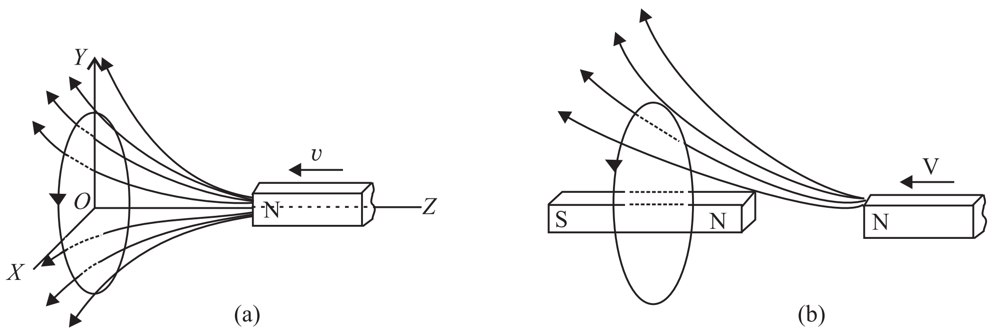


Fig.19.5: a) A bar magnet approaching a metal ring, and **b)** the magnetic field of the induced current opposes the approaching bar magnet.

The current induced in the ring creates a secondary magnetic field in it. This induced magnetic field can be taken as produced by a bar magnet, as shown in Fig.19.5 (b). Recall that induced magnetic field repels or opposes the original magnetic field. This opposition is a consequence of the law of conservation of energy, and is formalized as Lenz's law. **When a current is induced in a conductor, the direction of the current will be such that its magnetic effect opposes the change that induced it.**

The key word in the statement is 'oppose'-it tells us that we are not going to get something for nothing. When the bar magnet is pushed towards the ring, the current induced in the ring creates a magnetic field that opposes the change in flux. The magnetic field produced by the induced current repels the incoming magnet. If we wish to push the magnet towards the ring, we will have to do work on the magnet. This work shows up as electrical energy in the ring. Lenz's law thus follows from the law of conservation of energy. We can express the combined form of Faraday's and Lenz's laws as

$$\varepsilon = -\frac{d\phi}{dt} \tag{19.5}$$

The negative sign signifies opposition to the cause.

As an application of Lenz's law, let us reconsider the coil shown in Example 19.2. Suppose that its axis is chosen in vertical direction and the magnetic field is directed along it in upward direction. To an observer located directly above the coil, what would be the sense of the induced emf? It will be clockwise because only then the magnetic field due to it (directed downward by the right-hand rule) will oppose the changing magnetic flux. You should learn to apply Lenz's law before proceeding further. Try the following exercise.

19.1.3 Eddy currents

We know that the induced currents are produced in closed loops of conducting wires when the magnetic flux associated with them changes. However, induced currents are also produced when a solid conductor, usually in the form of a sheet or plate, is placed in a changing magnetic field. Actually, induced closed loops of currents are set up in the body of the conductor due to the change of flux linked with it. These currents flow in closed paths and in a direction perpendicular to the magnetic flux. These currents are called eddy currents as they look like eddies or whirlpools and also sometimes called Foucault currents as they were first discovered by Foucault.

The direction of these currents is given by Lenz's law according to which the direction will be such as to oppose the flux to which the induced currents are due. Fig. 19.1.3 shows some of the eddy currents in a metal sheet placed in an increasing magnetic field pointing into the plane of the paper. The eddy currents are circular and point in the anticlockwise direction.

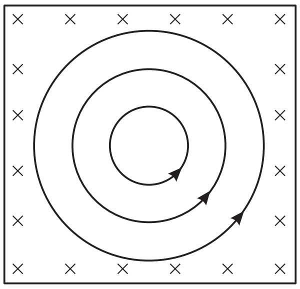


Fig. 19.1.3

The eddy currents produced in metallic bodies encounter little resistance and, therefore, have large magnitude. Obviously, eddy currents are considered undesirable in many electrical appliances and machines as they cause appreciable energy loss by way of heating. Hence, to reduce these currents, the metallic bodies are not taken in one solid piece but are rather made in parts or strips, called lamination, which are insulated from one another.

Eddy currents have also been put to some applications. For example, they are used in induction furnaces for making alloys of different metals in vacuum. They are also used in electric brakes for stopping electric trains.

INTEXT QUESTIONS 19.2

1. The bar magnet in Fig.19.6 moves to the right. What is the sense of the induced current in the stationary loop *A*? In loop *B*?

2. A cross-section of an ideal solenoid is shown in Fig.19.7. The magnitude of a uniform magnetic field is increasing inside the solenoid and **B** = 0 outside the solenoid. which conducting loops is there an induced current? What is the sense of the current in each case?

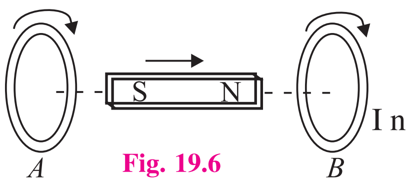


Fig. 19.6

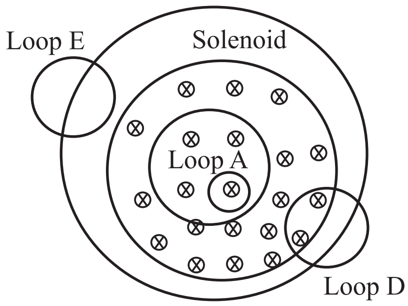


Fig. 19.7

3. A bar magnet, with its axis aligned along the axis of a copper ring, is moved along its length toward the ring. Is there an induced current in the ring? Is there an induced electric field in the ring? Is there a magnetic force on the bar magnet? Explain.

4. Why do we use laminated iron core in a transformer.

19.2 INDUCTANCE

When current in a circuit changes, a changing magnetic field is produced around it. If a part of this field passes through the circuit itself, current is induced in it. Now suppose that another circuit is brought in the neighbourhood of this circuit. Then the magnetic field through that circuit also changes, inducing an emf across it. Thus, induced emfs can appear in these circuits in two ways:

- By changing current in a coil, the magnetic flux linked with each turn of the coil changes and hence an induced emf appears across that coil. This property is called ***self-induction***.
- for a pair of coils situated close to each other such that the flux associated with one coil is linked through the other, a changing current in one coil induces an emf in the other. In this case, we speak of ***mutual induction*** of the pair of coils.

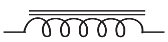
19.2.1 Self-Inductance

Let us consider a loop of a conducting material carrying electric current. The current produces a magnetic field **B**. The magnetic field gives rise to magnetic flux. The total magnetic flux linking the loop is

$$d\phi = \mathbf{B} . \mathbf{ds}$$

In the absence of any external source of magnetic flux (for example, an adjacent coil carrying a current), the Biot-Savart's law tells us that the magnetic field and hence flux will be proportional to the current (I) in the loop, i.e.

$$\phi \propto I \quad \text{or} \quad \phi = LI \tag{19.6}$$

where L is called self-inductance of the coil. The circuit elements which oppose change in current are called ***inductors.*** These are in general, in the form of coils of varied shapes and sizes. The symbol for an ***inductor*** is . If the coil is wrapped around an iron core so as to enhance its magnetic effect, it is symbolised by putting two lines above it, as shown here . The inductance of an indicator depends on its geometry.

(a) Faraday's Law in terms of Self-Inductance: So far you have learnt that if current in a loop changes, the magnetic flux linked through it also changes and gives rise to self–induced emf between the ends. In accordance with Lenz's law, the self-induced emf opposes the change that produces it.

To express the combined form of Faraday's and Lenz's Laws of induction in terms of L, we combine Eqns. (19.5) and (19.6) to obtain

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \tag{19.7a}$$

$$= -L\left(\frac{I_2 - I_1}{t}\right) \tag{19.7b}$$

where I_1 and I_2 respectively denote the initial and final values of current at $t = 0$ and $t = \tau$. Using Eqn. (19.7b), we can define the unit of self-inductance:

$$\text{units of } L = \frac{\text{unit of emf}}{\text{units of } dI/dt}$$

$$= \frac{\text{volt}}{\text{ampere / second}}$$

$$= \text{ohm-second}$$

An ohm-second is called a *henry*, (abbreviated H). For most applications, henry is a rather large unit, and we often use millihenry, mH (10^{-3} H) and microhenry μH (10^{-6}H) as more convenient measures.

The self-induced emf is also called the ***back emf***. Eqn.(19.7a) tells us that the ***back emf in an inductor*** depends on the rate of change of current in it and ***opposes the change in current.*** Moreover, since an infinite emf is not possible, from Eq.(19.7b) we can say that an instantaneous change in the inductor current cannot occur. Thus, we conclude that ***current through an inductor cannot change instantaneously.***

The inductance of an inductor depends on its geometry. In principle, we can calculate the self-inductance of any circuit, but in practice it is difficult except for devices with simple geometry. A solenoid is one such device used widely in electrical circuits as inductor. Let us calculate the self-inductance of a solenoid.

(b) Self-inductance of a solenoid : Consider a long solenoid of cross-sectional area A and length ℓ, which consists of N turns of wire. To find its inductance, we must relate the current in the solenoid to the magnetic flux through it. In the preceding lesson, you used Ampere's law to determine magnetic field of a long solenoid:

$$|\mathbf{B}| = \mu_0 n I$$

where $n = N/\ell$ denotes is the number of turns per unit length and I is the current through the solenoid.

The total flux through N turns of the solenoid is

$$\phi = N|\mathbf{B}|A = \frac{\mu_0 N^2 A I}{\ell} \qquad (19.8)$$

and self-inductance of the solenoid is

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{\ell} \qquad (19.9)$$

Using this expression, you can calculate self-inductance and back emf for a typical solenoid to get an idea of their magnitudes.

INTEXT QUESTIONS 19.3

1. A solenoid 1m long and 20cm in diameter contains 10,000 turns of wire. A current of 2.5A flowing in it is reduced steadily to zero in 1.0ms. Calculate the magnitude of back emf of the inductor while the current is being reduced.

2. A certain length (ℓ) of wire, folded into two parallel, adjacent strands of length $\ell/2$, is wound on to a cylindrical insulator to form a type of wire-wound non-inductive resistor (Fig.19.8). Why is this configuration called non-inductive?

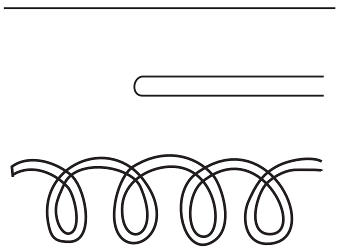


Fig.19.8: Wire wound on a cylindrical insulator

3. What rate of change of current in a 9.7 mH solenoid will produce a self-induced emf of 35mV?

19.2.2 LR Circuits

Suppose that a solenoid is connected to a battery through a switch (Fig.19.9). Beginning at $t = 0$, when the switch is closed, the battery causes charges to move in the circuit. *A* solenoid has inductance (L) and resistance (R), and each of these influence the current in the circuit. The inductive and resistive effects of a solenoid are shown schematically in Fig.19.10. The inductance (L) is shown in series with the resistance (R). For simplicity, we assume that total resistance in the circuit, including the internal resistance of the battery, is represented by R. Similarly, L includes the self-inductance of the connecting wires. A circuit such as that shown in Fig.19.9, containing resistance and inductance in series, is called an *LR* circuit.

The role of the inductance in any circuit can be understood qualitatively. As the current $i(t)$ in the circuit increases (from $i = 0$ at $t = 0$), a self-induced emf $\varepsilon = -L\,di/dt$ is produced in the inductance whose sense is opposite to the sense of the increasing current. This opposition to the increase in current prevents the current from rising abruptly.

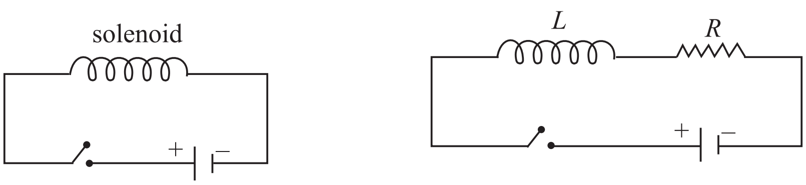


Fig. 19.9: LR Circuit

If there been no inductance in the circuit, the current would have jumped immediately to the maximum value defined by ε_0/R. But due to an inductance coil in the circuit, the current rises gradually and reaches a steady state value of ε_0/R as $t \to \tau$. The time taken by the current to reach about two-third of its steady state value is equal to by L/R, which is called the ***inductive time constant*** of the circuit. Significant changes in current in an LR circuit cannot occur on time scales much shorter that L/R. The plot of the current with time is shown in Fig. 19.10.

You can see that greater the value of L, the larger is the back emf, and longer it takes the current to build up. (This role of an inductance in an electrical circuit is somewhat similar to that of mass in mechanical systems.) That is why while switching off circuits cortaining large inductors, you should the mindful of back emf. The spark seen while turning off a switch connected to an electrical appliance such as a fan, computer, geyser or an iron, essentially arises due to back emf.

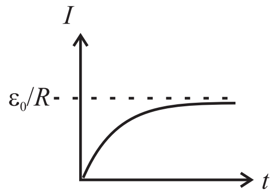


Fig.19.10 : Variation of current with time in a LR–ciruit.

INTEXT QUESTIONS 19.4

1. A light bulb connected to a battery and a switch comes to full brightness almost instantaneously when the switch is closed. However, if a large inductance is in series with the bulb, several seconds may pass before the bulb achieves full brightness. Explain why.

2. In an LR circuit, the current reaches 48mA in 2.2 ms after the switch is closed. After sometime the current reaches it steady state value of 72mA. If the resistance in the circuit is 68Ω, calculate the value of the inductance.

19.2.3 Mutual Inductance

When current changes in a coil, a changing magnetic flux develops around it, which may induce emf across an adjoining coil. As we see in Fig. (19.11), the magnetic flux linking each turn of coil B is due to the magnetic field of the current in coil A.

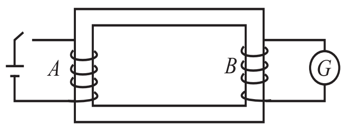


Fig. 19.11 : Mulual inductance of a pair of coils

Therefore, a changing current in each coil induces an emf in the other coil, i.e.

i.e.,
$$\phi_2 \propto \phi_1 \propto I_1 \Rightarrow \phi_2 = MI_1 \qquad (19.10)$$

Notes

where M is called the mutual inductance of the pair of coils. Also back emf induced across the second coil

$$e_2 = -\frac{d\phi}{dt}$$

$$= -M\frac{dI}{dt} = -M\left(\frac{I_2 - I_1}{t}\right) \qquad (19.11)$$

where the curent in coil A changes from I_1 to I_2 in t seconds.

The mutual inductance depends only on the geometry of the two coils, if no magnetic materials are nearby. The SI unit of mutual inductance is also henry (H), the same as the unit of self-inductance.

Example 19.3 : A coil in one circuit is close to another coil in a separate circuit. The mutual inductance of the combination is 340 mH. During a 15 ms time interval, the current in coil 1 changes steadily from 28mA to 57 mA and the current in coil 2 changes steadily from 36 mA to 16 mA. Determine the emf induced in each coil by the changing current in the other coil.

Solution : During the 15ms time interval, the currents in the coils change at the constant rates of

$$\frac{di_1}{dt} = \frac{57\text{mA} - 23\text{mA}}{15\text{ms}} = 2.3 \text{ As}^{-1}$$

$$\frac{di_2}{dt} = \frac{16\text{mA} - 36\text{mA}}{15\text{ms}} = -1.3 \text{ As}^{-1}$$

From Eq. (19.11), we note that the magnitudes of the induced emfs are

$$\varepsilon_1 = -\,(340\text{mH})\,(2.3\text{As}^{-1}) = -\,0.78 \text{ V}$$

$$\varepsilon_2 = (340\text{mH})\,(1.3\text{As}^{-1}) = 0.44 \text{ V}$$

Remember that the minus signs in Eq. (19.11) refer to the sense of each induced emf.

One of the most important applicances based on the phenomenon of mutual inductance is transformer. You will learn about it later in this lesson. Some commonly used devices based on self-inductance are the choke coil and the ignition coil. We will discuss about these devices briefly. Later, you will also learn that a combination of inductor and capitator acts as a basic oscillator. Once the capacitor is charged, the charge in this arrangement oscillates between its two plates through the inductor.