

Assignments in Mathematics Class X (Term I)

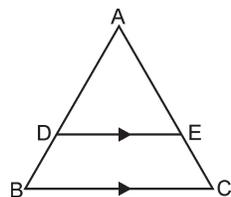
6. TRIANGLES

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- Two figures having the same shape but not necessarily the same size are called similar figures.
- All the congruent figures are similar but the converse is not true.
- Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).
- Two triangles are similar, if
 - (i) their corresponding angles are equal
 - (ii) their corresponding sides are in the same ratio (or proportion).

- **Basic Proportionality Theorem (B.P.T.) (Thales Theorem)**

In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio.

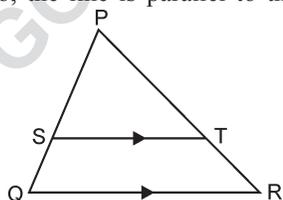


In $\triangle ABC$, if $DE \parallel BC$ then (i) $\frac{AD}{DB} = \frac{AE}{EC}$

(ii) $\frac{AB}{AD} = \frac{AC}{AE}$ (iii) $\frac{AB}{DB} = \frac{AC}{EC}$

- **Converse of Basic Proportionality Theorem**

If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

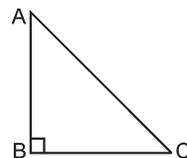


In $\triangle PQR$, if $\frac{PS}{SQ} = \frac{PT}{TR}$, then $ST \parallel QR$.

- If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (*AAA similarity criterion*).

- If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (*AA similarity criterion*).
- If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (*SSS similarity criterion*).
- If one angle of a triangle is equal to the one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (*SAS similarity criterion*).
- If a perpendicular is drawn from the vertex of the right angle of a right triangle to hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
- The ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.
- The areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.
- The areas of two similar triangles are in the ratio of the squares of the corresponding medians.
- If the areas of two similar triangles are equal, then the triangles are congruent, i.e., equal and similar triangles are congruent.
- **The Pythagoras Theorem**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In figure, $\angle B = 90^\circ$, so, $AC^2 = AB^2 + BC^2$.



- **Converse of the Pythagoras Theorem**

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

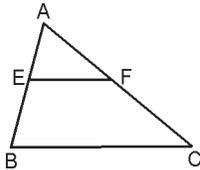
SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS

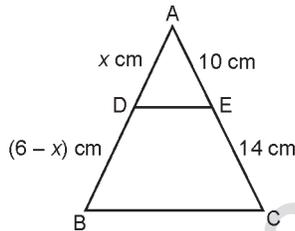
[1 Mark]

A. Important Questions

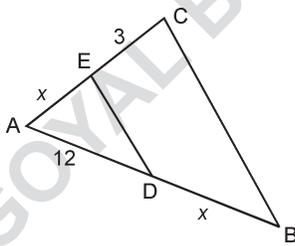
1. In the figure, if $\frac{AE}{EB} = \frac{AF}{FC}$, then we can conclude that :



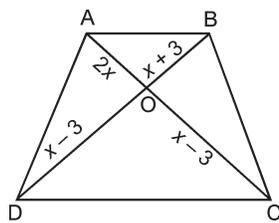
- (a) E and F are the mid-points of AB and AC respectively
 (b) $EF \parallel BC$
 (c) $\frac{EF}{BC} = \frac{AB}{AC}$
 (d) none of the above
2. In the triangle ABC , $DE \parallel BC$, then the length of DB is :



- (a) 2.5 cm (b) 5 cm (c) 3.5 cm (d) 3 cm
3. In $\triangle ABC$, if $DE \parallel BC$, then the value of x is :

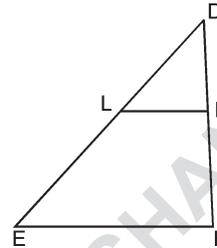


- (a) 4 (b) 6 (c) 8 (d) 9
4. In the trapezium $ABCD$, $AB \parallel CD$, then the value of x is :

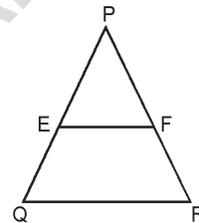


- (a) 2 (b) 3 (c) -2 (d) -3

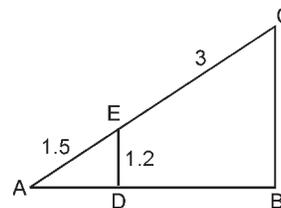
5. In the $\triangle DEF$, $LM \parallel EF$ and $\frac{DM}{MF} = \frac{2}{3}$. If $DE = 5.5$ cm, then DL is :



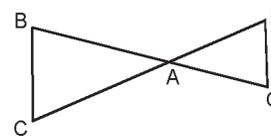
- (a) 2.5 cm (b) 2.4 cm (c) 2.2 cm (d) 2 cm
6. In the given figure, $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm, then :



- (a) EF is not parallel to QR
 (b) $EF \parallel QR$
 (c) cannot say anything
 (d) none of the above
7. In the given figure, if $\triangle ADE \sim \triangle ABC$, then BC is equal to :



- (a) 4.5 (b) 3 (c) 3.6 (d) 2.4
8. In the given figure. $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm, then the length of AQ is :



- (a) 3.25 cm (b) 4 cm (c) 4.25 cm (d) 3 cm
9. If $\triangle ABC \sim \triangle PQR$ and $\angle P = 50^\circ$, $\angle B = 60^\circ$, then $\angle R$ is :

- (a) 100° (b) 80° (c) 70° (d) cannot be determined

10. $\triangle ABC \sim \triangle DEF$ and the perimeters of $\triangle ABC$ and $\triangle DEF$ are 30 cm and 18 cm respectively.

If $BC = 9$ cm, then EF is equal to :

- (a) 6.3 cm (b) 5.4 cm (c) 7.2 cm (d) 4.5 cm

11. $\triangle ABC \sim \triangle DEF$ such that $AB = 9.1$ cm and $DE = 6.5$ cm. If the perimeter of $\triangle DEF$ is 25 cm, then perimeter of $\triangle ABC$ is :

- (a) 35 cm (b) 28 cm (c) 42 cm (d) 40 cm

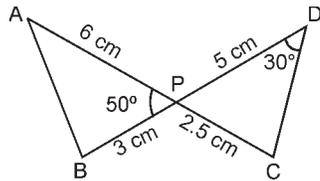
12. If $\triangle ABC \sim \triangle EDF$ and $\triangle ABC$ is not similar to $\triangle DEF$, then which of the following is not true?

- (a) $BC \cdot EF = AC \cdot FD$ (b) $AB \cdot EF = AC \cdot DE$
(c) $BC \cdot DE = AB \cdot EF$ (d) $BC \cdot DE = AB \cdot FD$

13. If in two triangles ABC and PQR , $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then :

- (a) $\triangle PQR \sim \triangle CAB$ (b) $\triangle PQR \sim \triangle ABC$
(c) $\triangle CBA \sim \triangle PQR$ (d) $\triangle BCA \sim \triangle PQR$

14. In the given figure, two line segments, AC and BD intersect each other at the point P such that $PA = 6$ cm, $PB = 3$ cm, $PC = 2.5$ cm, $PD = 5$ cm, $\angle APB = 50^\circ$ and $\angle CDP = 30^\circ$. Then $\angle PBA$ is equal to:



- (a) 50° (b) 30° (c) 60° (d) 100°

15. If in triangles ABC and DEF , $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when :

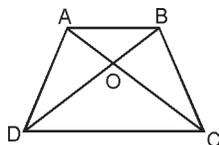
- (a) $\angle B = \angle E$ (b) $\angle A = \angle D$
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

16. The areas of two similar triangles are 169 cm^2 and 121 cm^2 , if the longest side of the larger triangle is 26 cm, then the longest side of the other triangle is :

- (a) 12 cm (b) 14 cm (c) 19 cm (d) 22 cm

17. In the following trapezium $ABCD$, $AB \parallel CD$ and $CD = 2AB$. If area $(\triangle AOB) = 84 \text{ cm}^2$, then area $(\triangle COD)$ is :

- (a) 168 cm^2 (b) 336 cm^2
(c) 252 cm^2 (d) none of these



18. If $\triangle ABC \sim \triangle PQR$, area $(\triangle ABC) = 80 \text{ cm}^2$ and area $(\triangle PQR) = 245 \text{ cm}^2$, then $\frac{AB}{PQ}$ is equal to :

- (a) 16 : 49 (b) 4 : 7
(c) 2 : 5 (d) none of these

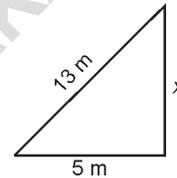
19. In the similar triangles, $\triangle ABC$ and $\triangle DEF$, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{3}{4}$. If the median $AL = 6$ cm, then

the median DM of $\triangle DEF$ is :

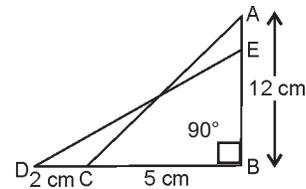
- (a) $3\sqrt{2} \text{ cm}$ (b) $4\sqrt{3} \text{ cm}$
(c) $4\sqrt{2} \text{ cm}$ (d) $3\sqrt{3} \text{ cm}$

20. If a ladder of length 13 m is placed against a wall such that its foot is at a distance of 5 m from the wall, then the height of the top of the ladder from the ground is :

- (a) 10 m (b) 11 m
(c) 12 m (d) none of these

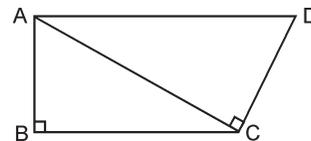


21. In the figure, if $AC = DE$, then the value of EB is :



- (a) $3\sqrt{30} \text{ cm}$ (b) $2\sqrt{30} \text{ cm}$
(c) $3\sqrt{15} \text{ cm}$ (d) $4\sqrt{15} \text{ cm}$

22. In the quadrilateral $ABCD$, if $\angle B = 90^\circ$ and $\angle ACD = 90^\circ$, then AD^2 is:

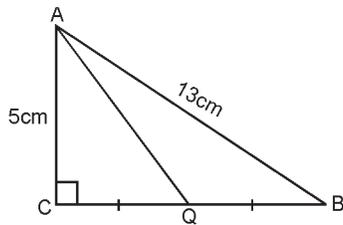


- (a) $AC^2 - AB^2 + BC^2$ (b) $AC^2 + DC^2 + AB^2$
(c) $AB^2 + BC^2 + CD^2$ (d) $AB^2 + BC^2 + AC^2$

23. If diagonals of a rhombus are 12 cm and 16 cm, then the perimeter of the rhombus is :

- (a) 20 cm (b) 40 cm (c) 28 cm (d) 56 cm

24. In the figure, $\triangle ABC$ is right angled at C and Q is the mid-point of BC , then the length of AQ is :



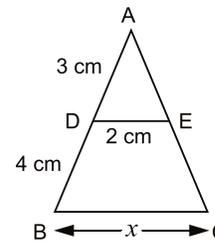
- (a) 6 cm (b) 12 cm
(c) $\sqrt{61}$ cm (d) $6\sqrt{3}$ cm
25. In triangle ABC and DEF , $\angle A \neq \angle C$, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = EF$. Then, the two triangles are :
- (a) neither congruent nor similar
(b) congruent as well as similar
(c) congruent but not similar
(d) similar but not congruent
26. If in triangle ABC and DEF , $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, if :

- (a) $\angle B = \angle E$ (b) $\angle A = \angle D$
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

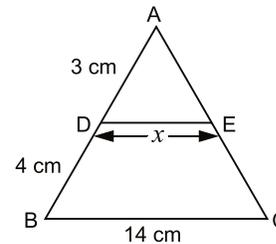
27. If $\triangle ABC \sim \triangle QRP$, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{9}{4}$, perimeter $\triangle ABC = 48$ cm, $AB = 18$ cm and $BC = 18$ cm, then PQ is equal to :
- (a) 8 cm (b) 10 cm (c) 12 cm (d) $\frac{20}{3}$ cm
28. D and E are respectively the points on the sides AB and AC of a triangle ABC such that $AD = 2$ cm, $BD = 4$ cm, $BC = 9$ cm and $DE \parallel BC$. Then, length of DE (in cm) is :
- (a) 6 (b) 5 (c) 3 (d) 2.5
29. It is given that $\triangle ABC \sim \triangle PQR$, with $\frac{BC}{QR} = \frac{1}{4}$, then, $\frac{\text{ar}(PQR)}{\text{ar}(BCA)}$ is equal to :
- (a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) 4 (d) 16

B. Questions From CBSE Examination Papers

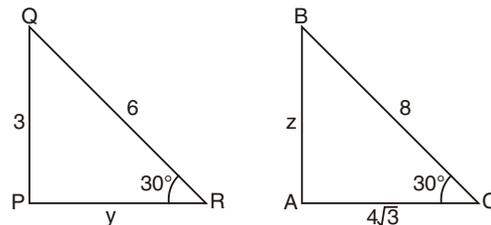
1. The lengths of the diagonals of a rhombus are 24 cm and 32 cm. The perimeter of the rhombus is : [2010 (T-I)]
(a) 9 cm (b) 128 cm (c) 80 cm (d) 56 cm
2. Which of the following cannot be the sides of a right triangle ? [2010 (T-I)]
(a) 9 cm, 15 cm, 12 cm
(b) 2 cm, 1 cm, $\sqrt{5}$ cm
(c) 400 mm, 300 mm, 500 mm
(d) 9 cm, 5 cm, 7 cm
3. $\triangle ABC \sim \triangle PQR$, M is the mid-point of BC and N is the mid point of QR . If the area of $\triangle ABC = 100$ sq. cm, the area of $\triangle PQR = 144$ sq. cm and $AM = 4$ cm, then PN is : [2010 (T-I)]
(a) 4.8 cm (b) 12 cm (c) 4 cm (d) 5.6 cm
4. $\triangle ABC$ is such that $AB = 3$ cm, $BC = 2$ cm and $CA = 2.5$ cm. If $\triangle DEF \sim \triangle ABC$ and $EF = 4$ cm, then perimeter of $\triangle DEF$ is : [2010 (T-I)]
(a) 15 cm (b) 22.5 cm (c) 7.5 cm (d) 30 cm
5. A vertical stick 30 m long casts a shadow 15 m long on the ground. At the same time, a tower casts a shadow 75 m long on the ground. The height of the tower is : [2010 (T-I)]
(a) 150 m (b) 100 m (c) 25 m (d) 200 m
6. $\triangle ABC \sim \triangle PQR$. If $\text{ar}(ABC) = 2.25$ m², $\text{ar}(PQR) = 6.25$ m², $PQ = 0.5$ m, then length of AB is : [2010 (T-I)]
(a) 30 cm (b) 0.5 m (c) 50 m (d) 3 m
7. In figure, if $DE \parallel BC$ then x equals to :



- (a) 3 cm (b) 4 cm (c) 7 cm (d) 4.7 cm
8. In figure, if $DE \parallel BC$, then x equals to : [2010 (T-I)]



- (a) 6 cm (b) 7 cm (c) 3 cm (d) 4 cm
9. $\triangle ABC$ and $\triangle PQR$ are similar triangles such that $\angle A = 32^\circ$ and $\angle R = 65^\circ$, then $\angle B$ is : [2010 (T-I)]
(a) 83° (b) 32° (c) 65° (d) 97°
10. In the figure $\triangle ABC \sim \triangle PQR$, then $y + z$ is :



- (a) $2 + \sqrt{3}$ (b) $4 + 3\sqrt{3}$ (c) $4 + \sqrt{3}$ (d) $3 + 4\sqrt{3}$
11. The perimeters of two similar triangles ABC and LMN are 60 cm and 48 cm respectively.

[2010 (T-I)]

If $LM = 8$ cm, length of AB is :

- (a) 10 cm (b) 8 cm (c) 5 cm (d) 6 cm
12. If in $\triangle ABC$ and $\triangle DEF$ $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar if :
- (a) $\angle B = \angle E$ (b) $\angle A = \angle D$
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$
13. In an isosceles $\triangle ABC$, if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C$ is equal to :
- (a) 45° (b) 60° (c) 30° (d) 90°
14. If $\triangle ABC \sim \triangle DEF$, $BC = 4$ cm, $EF = 5$ cm and $ar(\triangle ABC) = 80$ cm², then $ar(\triangle DEF)$ is :

[2010 (T-I)]

- (a) 100 cm² (b) 125 cm²
(c) 150 cm² (d) 200 cm²

15. The areas of two similar triangles ABC and PQR are 25 cm² and 49 cm² respectively. If $QR = 9.8$ cm, then BC is :

[2010 (T-I)]

- (a) 9.8 cm (b) 7 cm (c) 49 cm (d) 25 cm

16. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then the ratio of their corresponding altitude is :

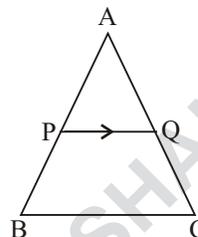
[2010 (T-I)]

- (a) 3 : 2 (b) 16 : 81 (c) 4 : 9 (d) 2 : 3

17. In the figure, $PQ \parallel BC$ and $AP : PB = 1 : 2$. Find

$$\frac{ar(\triangle APQ)}{ar(\triangle ABC)} :$$

[2010 (T-I)]



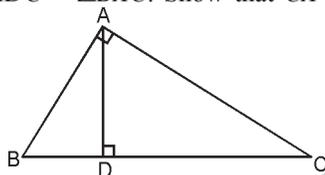
- (a) 1 : 4 (b) 4 : 1 (c) 1 : 9 (d) 2 : 9

SHORT ANSWER TYPE QUESTIONS

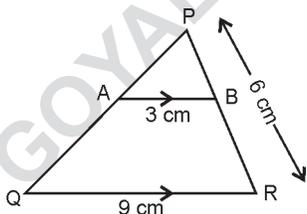
[2 Marks]

A. Important Questions

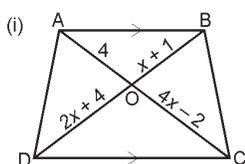
1. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.



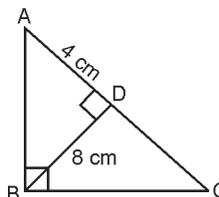
2. S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.
3. In the given figure, $AB \parallel QR$. Find the length of PB .



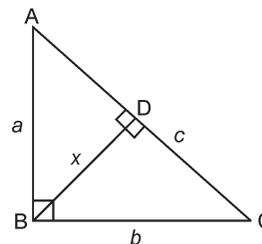
4. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is a right-angled triangle.
5. In the given figure, if $AB \parallel CD$, find the value of x .



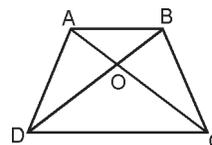
6. In the given figure, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm and $AD = 4$ cm, find CD .



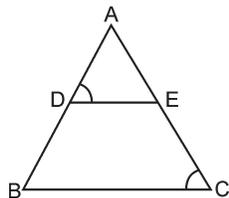
7. In a right angled triangle with sides a and b and hypotenuse c , the altitude drawn on the hypotenuse is x . Prove that $ab = cx$.



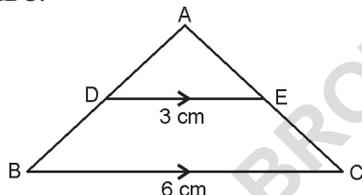
8. In the figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 8$ cm. Find the value of DC .



9. In the given figure, $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle ADE = \angle ACB$. Prove that $\triangle ABC$ is an isosceles triangle.

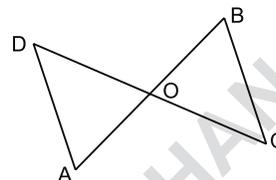


10. E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F . Show that $\triangle ABE \sim \triangle CFB$.
11. P and Q are points on sides AB and AC respectively of $\triangle ABC$. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3 PQ$.
12. In a triangle ABC , altitudes AL and BM intersect in O . Prove that $\frac{AO}{BO} = \frac{OM}{OL}$.
13. A wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?
14. In the given figure, $DE \parallel BC$. If $DE = 3$ cm, $BC = 6$ cm and area $(\triangle ADE) = 15$ cm², find the area of $\triangle ABC$.



15. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
16. $ABCD$ is a trapezium with $AB \parallel DC$. If E and F are on non-parallel sides AD and BC respectively such that EF is parallel to AB , then show that $\frac{AE}{ED} = \frac{BF}{FC}$.

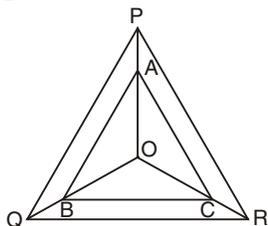
17. In the given figure, $OC \cdot OD = OA \cdot OB$. Show that $\angle A = \angle C$ and $\angle B = \angle D$.



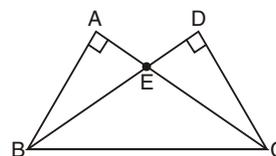
18. D , E and F are respectively the mid-points of sides AB , BC and CA of $\triangle ABC$. Then find ar $\triangle DEF$: ar $\triangle ABC$.
19. Prove that the area of the semicircle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle.
20. A ladder is placed against a wall such that its foot is at a distance of 3.5 m from the wall and its top reaches a window 12 m above the ground. Find the length of the ladder.
21. If ABC is an equilateral triangle of side $2a$, then find each of its altitude.
22. An aeroplane leaves an airport and flies due north at a speed of 500 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after 2 hours?

B. Questions From CBSE Examination Papers

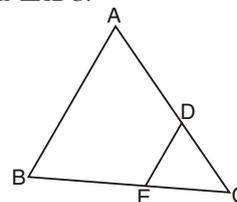
1. In the figure, A , B and C are points on OP , OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$. [2010 (T-I)]



2. In the figure, two triangles ABC and DBC are on the same base BC in which $\angle A = \angle D = 90^\circ$. If CA and BD meet each other at E , show that $AE \times CE = BE \times DE$. [2010 (T-I)]



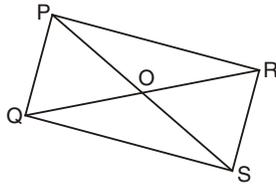
3. In the figure, if $\angle A = \angle B$ and $AD = BE$, show that $DE \parallel AB$ in $\triangle ABC$. [2010 (T-I)]



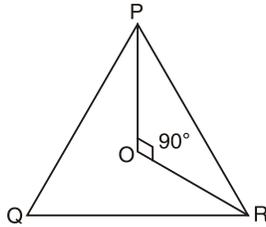
4. In the figure, PQR and SQR are two triangles on the same base QR . If PS intersect QR at O , then

show that : $\frac{\text{ar}(PQR)}{\text{ar}(SQR)} = \frac{PO}{SO}$.

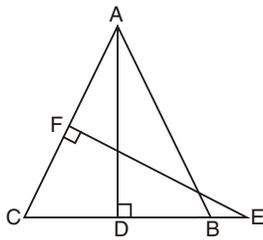
[2010 (T-I)]



5. In the figure, O is a point inside ΔPQR such that $\angle POR = 90^\circ$, $OP = 6$ cm and $OR = 8$. If $PQ = 24$ cm, $QR = 26$ cm, prove that ΔQPR is a right angled triangle. [2010 (T-I)]



6. In the given figure, E is a point on side CB produced of an isosceles ΔABC with $AB = BC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$. [2010 (T-I)]

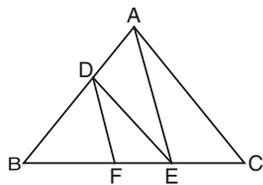


7. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal. [2010 (T-I)]

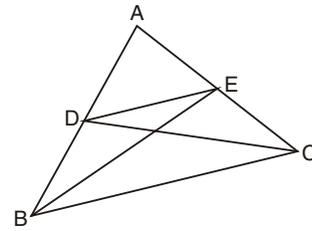
8. In a right angled triangle if hypotenuse is 20 cm and the ratio of other two sides is 4 : 3, find the sides. [2010 (T-I)]

9. In an isosceles triangle ABC , if $AB = AC = 13$ cm and the altitude from A on BC is 5 cm, find BC .

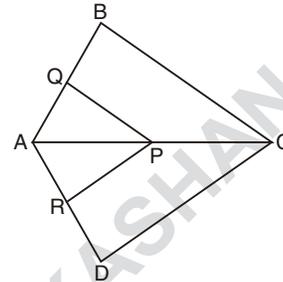
10. In the figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{EF}{BF} = \frac{EC}{BE}$. [2010 (T-I)]



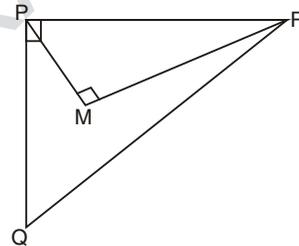
11. In the figure, $\Delta ABE \cong \Delta ACD$. Prove that $\Delta ADE \sim \Delta ABC$. [2010 (T-I)]



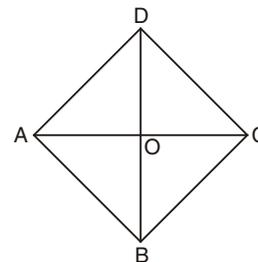
12. In the figure, if $PQ \parallel CB$ and $PR \parallel CD$, prove that $\frac{AR}{AD} = \frac{AQ}{AB}$. [2010 (T-I)]



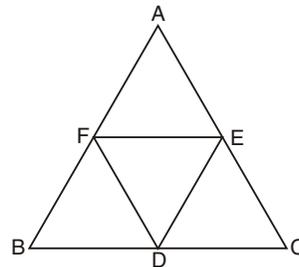
13. In the figure, $PM = 6$ cm, $MR = 8$ cm and $QR = 26$ cm, find the length of PQ . [2010 (T-I)]



14. In the figure, $ABCD$ is a rhombus. Prove that $4AB^2 = AC^2 + BD^2$. [2010 (T-I)]



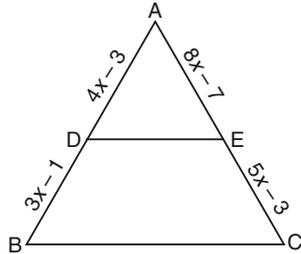
15. In the figure, D, E, F , are mid-points of sides BC, CA, AB respectively of ΔABC . Find the ratio of area of ΔDEF to area of ΔABC . [2010 (T-I)]



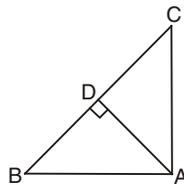
16. If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 2, prove that one of the parallel sides is double the other. [2010 (T-I)]

17. In $\triangle ABC$, $AB = AC$ and D is a point on side AC such that $BC^2 = AC \cdot CD$. Prove that $BD = BC$. [2010 (T-I)]

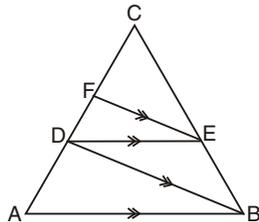
18. In the figure $DE \parallel BC$. Find x . [2010 (T-I)]



19. In the figure, $\angle BAC = 90^\circ$, $AD \perp BC$. Prove that $AB^2 + CD^2 = BD^2 + AC^2$. [2010 (T-I)]



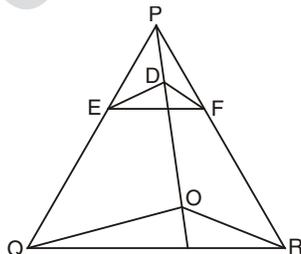
20. In the figure, $AB \parallel DE$ and $BD \parallel EF$. Prove that $DC^2 = CF \times AC$. [2010 (T-I)]



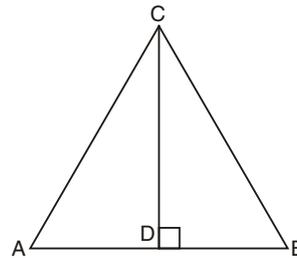
21. If D, E are points on the sides AB and AC of a $\triangle ABC$ such that $AD = 6$ cm, $BD = 9$ cm, $AE = 8$ cm $EC = 12$ cm. Prove that $DE \parallel BC$. [2010 (T-I)]

22. Prove that the line joining the mid points of any two sides of a triangle is parallel to the third side. [2010 (T-I)]

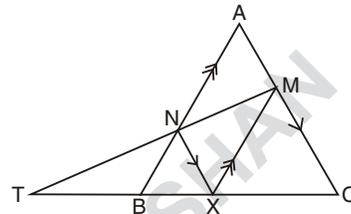
23. In the figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$. [2010 (T-I)]



24. In the figure, $\angle ACB = 90^\circ$ and $CD \perp AB$. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$. [2010 (T-I)]

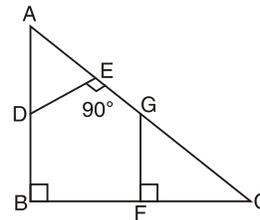


25. In the figure, $XN \parallel CA$ and $XM \parallel BA$. T is a point on CB produced. Prove that $TX^2 = TB \cdot TC$. [2010 (T-I)]



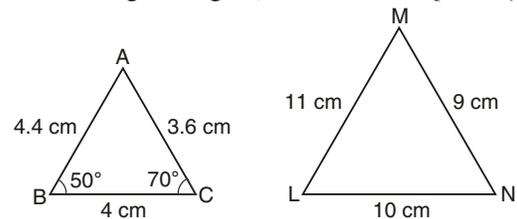
26. Two poles of height 10 m and 15 m stand vertically on a plane ground. If the distance between their feet is $5\sqrt{3}$ m, find the distance between their tops. [2010 (T-I)]

27. In the figure, $AB \perp BC$, $DE \perp AC$ and $GF \perp BC$. Prove that $\triangle ADE \sim \triangle GCF$. [2010 (T-I)]

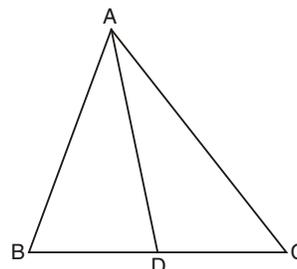


28. In $\triangle ABC$, $AD \perp BC$ such that $AD^2 = BD \times CD$. Prove that $\triangle ABC$ is right angled at A . [2010 (T-I)]

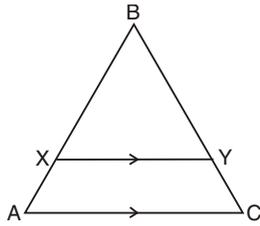
29. From the given figure, find $\angle MLN$. [2010 (T-I)]



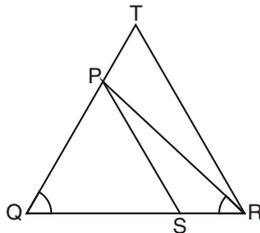
30. In a quadrilateral $ABCD$, $\angle B = 90^\circ$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^\circ$.



31. In the figure, D is point on the side BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$.

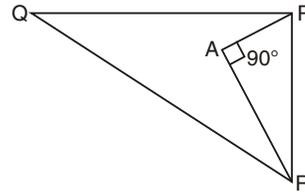


32. In the figure, $XY \parallel AC$ and XY divides triangular region ABC into two parts equal in area. Find the ratio of $\frac{AX}{AB}$. [2010 (T-I)]
33. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right angled triangle. [2010 (T-I)]

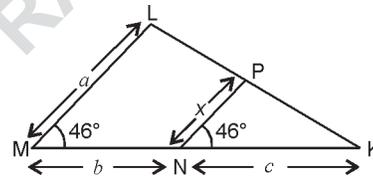


34. In the figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle TQR = \angle PRS$. Show that $\triangle PQS \sim \triangle PQR$. [2010 (T-I)]
35. D, E and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Determine the ratio of the areas of $\triangle DEF$ and $\triangle ABC$. [2008]

36. In the given figure, $PQ = 24$ cm, $QR = 26$ cm, $\angle PAR = 90^\circ$, $PA = 6$ cm and $AR = 8$ cm. Find $\angle QPR$. [2008]



37. E is a point on the side AD produced of a \parallel gm $ABCD$ and BE intersects CD at F . Show that $\triangle ABE \sim \triangle CFB$. [2008]
38. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus. [2008]
39. A man goes 15 meters due west and then 8 metres due north. How far is he from the starting point? [2008]
40. In the figure, $\angle M = \angle N = 46^\circ$. Express x in terms of a, b and c , where a, b and c are lengths of LM, MN and NK respectively. [2009]



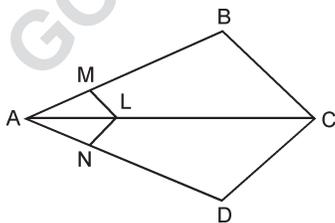
41. In a $\triangle ABC$, $DE \parallel BC$. If $DE = \frac{2}{3} BC$ and area of $\triangle ABC = 81 \text{ cm}^2$, find the area of $\triangle ADE$. [2009]

SHORT ANSWER TYPE QUESTIONS

[3 Marks]

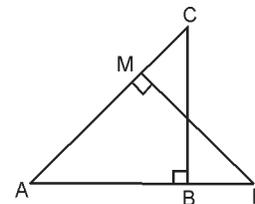
A. Important Questions

1. In the figure, $LM \parallel CB$ and $LN \parallel CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.

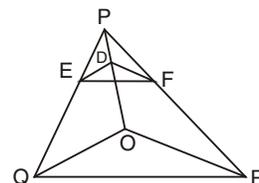


2. The diagonals of a trapezium divide each other in the same ratio. Prove
3. In the given figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that :
(i) $\triangle ABC \sim \triangle AMP$

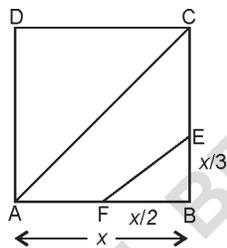
(ii) $\frac{CA}{PA} = \frac{BC}{MP}$.



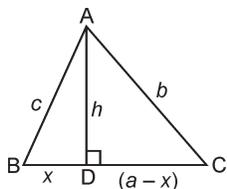
4. In the figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.



5. The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that $ABCD$ is a trapezium.
6. By using the converse of the basic proportionality theorem, show that the line joining mid-points of non-parallel sides of a trapezium is parallel to the parallel sides.
7. If three or more parallel lines are intersected by two transversals, the intercepts made by them on the transversals are proportional. Prove.
8. PQR is triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM.MR$.
9. $ABCD$ is a trapezium in which $AB \parallel CD$. The diagonals AC and BD intersect at O . Prove that : (i) $\triangle AOB \sim \triangle COD$ (ii) If $OA = 6$ cm and $OC = 8$ cm, find : $\frac{\text{area}(\triangle ABO)}{\text{area}(\triangle COD)}$.
10. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C . Prove that $AE^2 + BD^2 = AB^2 + DE^2$.
11. ABC is a triangle in which $\angle A = 90^\circ$, $AN \perp BC$, $AB = 12$ cm and $AC = 5$ cm. Find the ratio of the areas of $\triangle ANC$ and $\triangle ABC$.
12. $ABCD$ is a square. F is the mid-point of AB . BE is the one-third of BC . If the area of the $\triangle BFE$ is 108 cm², find the length of AC . **[HOTS]**



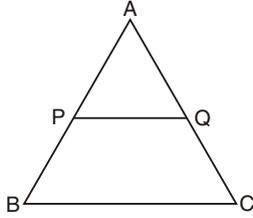
13. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?
14. In the given figure, $\angle B < 90^\circ$ and segment $AD \perp BC$, show that $b^2 = h^2 + a^2 + x^2 - 2ax$



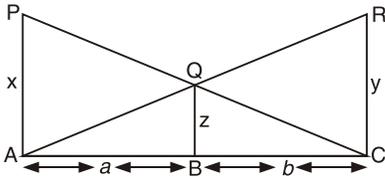
15. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.
16. Using BPT, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.
17. Using converse of BPT, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.
18. Line l drawn parallel to side AB of quadrilateral $ABCD$ meets AD at E and BC at F such that $\frac{AE}{ED} = \frac{BF}{FC}$. Then show that $ABCD$ is a trapezium.
19. A girl of height 120 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 4.8 m above the ground, find the length of her shadow after 4 seconds.
20. If $\triangle ABC \sim \triangle DEF$ and, AL and DM are their corresponding medians, then show that $\frac{AL}{DM} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
21. If $\triangle ABC \sim \triangle DEF$ and, AL and DM are their corresponding altitudes, then show that $\frac{AL}{DM} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
22. If $\triangle ABC \sim \triangle DEF$, then show that $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
23. Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O . If $AB = 3CD$, find the ratio of the areas of triangles AOB and COD .
24. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.
25. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding altitudes.
26. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding perimeters.

B. Questions From CBSE Examination Papers

1. In $\triangle ABC$, in figure, PQ meets AB in P and AC in Q . If $AP = 1$ cm, $PB = 3$ cm, $AQ = 1.5$ cm, $QC = 4.5$ cm, prove that area of $\triangle APQ$ is one sixteenth of the area of $\triangle ABC$. [2010 (T-I)]



2. In the figure, PA , QB and RC are perpendiculars to AC . Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ [2010 (T-I)]



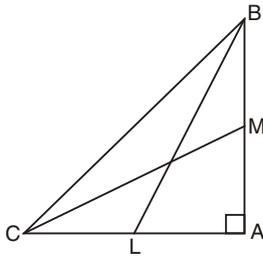
3. In an equilateral triangle ABC , D is a point on side BC such that $3BD = BC$. Prove that $9AD^2 = 7AB^2$ [2010 (T-I)]

4. In $\triangle PQR$, $PD \perp QR$ such that D lies on QR . If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$ and a, b, c, d are positive units, prove that $(a + b)(a - b) = (c + d)(c - d)$. [2010 (T-I)]

5. In $\triangle ABC$, $AD \perp BC$ such that $AB^2 = BD \cdot CD$. Prove that ABC is a right triangle, right angle at A . [2010 (T-I)]

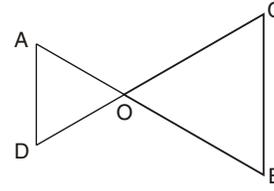
6. P and Q are points on sides AB and AC respectively of $\triangle ABC$. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3PQ$. [2010 (T-I)]

7. In figure, BL and CM are medians of $\triangle ABC$ right angled at A . Prove that $4(BL^2 + CM^2) = 5BC^2$ [2010 (T-I)]



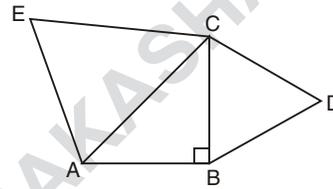
8. The perpendicular AD on the base BC of $\triangle ABC$ intersects BC in D such that $BD = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$. [2010 (T-I)]

9. In the figure, $OA \cdot OB = OC \cdot OD$. Show that $\angle A = \angle C$ and $\angle B = \angle D$. [2010 (T-I)]

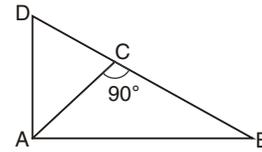


10. In the figure, ABC is an isosceles triangle right angled at B . Two equilateral triangles are constructed with side BC and AC . Prove that :

$$\text{ar } \triangle BCD = \frac{1}{2} \text{ar } \triangle ACE \quad [2010 (T-I)]$$



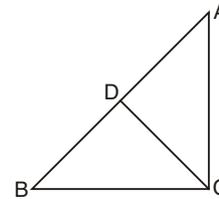
11. In the figure, ABD is a triangle in which $\angle DAB = 90^\circ$ and $AC \perp BD$. Prove that $AB^2 = BC \times BD$. [2010 (T-I)]



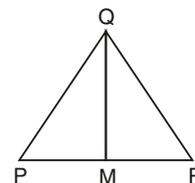
12. In ABC , if AD is the median, then show that $AB^2 + AC^2 = 2[AD^2 + BD^2]$. [2010 (T-I)]

13. In figure, ABC is right triangle right angled at C . Let $BC = a$, $CA = b$, $AB = c$ and let p be the length of perpendicular from C on AB . Prove that :

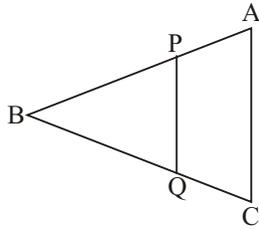
$$(i) cp = ab \quad (ii) \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} \quad [2010 (T-I)]$$



14. In the figure, PQR is a triangle in which $QM \perp PR$ and $PR^2 - PQ^2 = QR^2$, prove that $QM^2 = PM \times MR$. [2010 (T-I)]

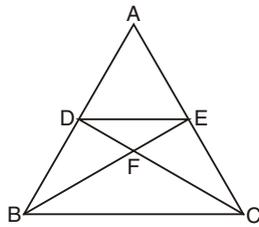


15. In figure, the line segment PQ is parallel to AC of triangle ABC and it divides the triangle into two parts of equal area. Find the ratio $\frac{AP}{AB}$.



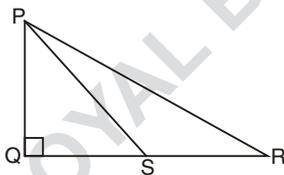
[2010 (T-I)]

16. In figure, $DE \parallel BC$ and $AD : DB = 5 : 1$. Find $\frac{\text{ar}(\triangle DFE)}{\text{ar}(\triangle CFB)}$.



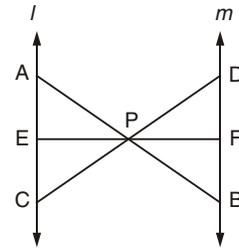
[2010 (T-I)]

17. Two isosceles triangles have equal vertical angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights. [2010 (T-I)]
18. Prove that the equilateral triangles described on the two sides of a right-angled triangle are together equal to the equilateral triangle described on the hypotenuse in terms of their areas. [2010 (T-I)]
19. In the figure, PQR is a right angled triangle in which $\angle Q = 90^\circ$. If $QS = SR$, show that $PR^2 - 4PS^2 = 3PQ^2$



[2010 (T-I)]

20. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. [2010 (T-I)]
21. If the areas of two similar triangles are equal, prove that they are congruent. [2010 (T-I)]
22. In an isosceles triangle ABC with $AB = AC$, $BD \perp AC$, prove that $BD^2 - CD^2 = 2CD \cdot AD$. [2010 (T-I)]
23. In the figure, $l \parallel m$ and line segments AB , CD and EF are concurrent at P . Prove that : $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$.



24. Triangle ABC is right angled at B and D is mid point of BC . Prove that : $AC^2 = 4AD^2 - 3AB^2$.

[2010 (T-I)]

25. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

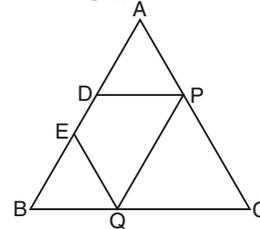
[2010 (T-I)]

26. In the figure, $\angle QPR = 90^\circ$, $\angle PMR = 90^\circ$, $QR = 26$ cm, $PM = 8$ cm, $MR = 6$ cm. Find area $(\triangle PQR)$.

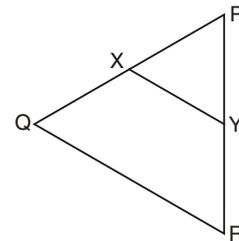
[2010 (T-I)]



27. In $\triangle ABC$, D and E are two points lying on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$. [2010 (T-I)]

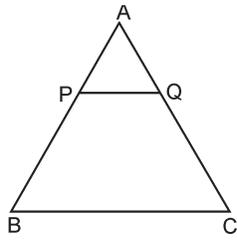


28. In figure, $XY \parallel QR$, $\frac{PQ}{XQ} = \frac{7}{3}$ and $PR = 6.3$ cm. Find YR . [2010 (T-I)]

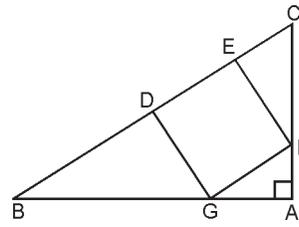


29. In $\triangle ABC$, AD is a median and E is mid-point of AD . If BE is produced, it meets AC at F . Show that $AF = \frac{1}{3}AC$. [2006]

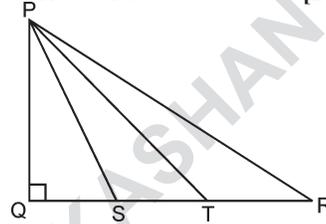
30. In the given figure, $PQ \parallel BC$ and $AP : PB = 1 : 2$. Find $\frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)}$. [2008]



31. If D is a point on the side AB of $\triangle ABC$ such that $AD : DB = 3 : 2$ and E is a point on BC such that $DE \parallel AC$. Find the ratio of areas of $\triangle ABC$ and $\triangle BDE$. [2001, 2006 C]
32. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. [2005, 2006]
33. In the figure, $DEFG$ is a square and $\angle BAC = 90^\circ$. Show that $DE^2 = BD \times EC$. [2009]



34. O is a point in the interior of rectangle $ABCD$. If O is joined to each of the vertices of the rectangle, prove that $OB^2 + OD^2 = OA^2 + OC^2$. [2006 C]
35. In the figure, $\triangle PQR$ is right angled at Q and the points S and T trisect the side QR . Prove that $8PT^2 = 3PR^2 + 5PS^2$ [2006 C, 2009]

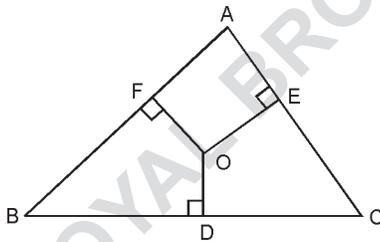


LONG ANSWER TYPE QUESTIONS

[4 Marks]

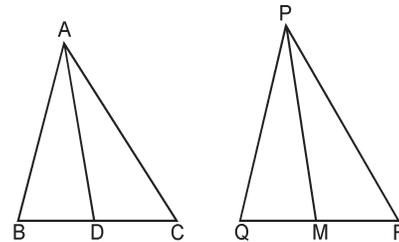
A. Important Questions

1. In the given figure, O is a point in the interior of a triangle ABC , $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that
- (i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$
- (ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$ [HOTS]

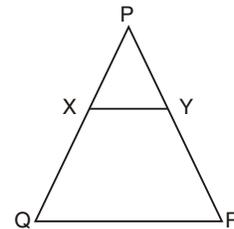


2. $ABCD$ is a quadrilateral. P, Q, R, S are the points of trisection of the sides AB, BC, CD and DA respectively. Prove that $PQRS$ is a parallelogram.
3. In a right triangle ABC , right angled at C . P and Q are points on the sides CA and CB respectively which divide these sides in the ratio $1 : 2$. Prove that :
- (i) $9AQ^2 = 9AC^2 + 4BC^2$
- (ii) $9BP^2 = 9BC^2 + 4AC^2$
- (iii) $9(AQ^2 + BP^2) = 13AB^2$
4. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR

and median PM of $\triangle PQR$ (see figure). Show that $\triangle ABC \sim \triangle PQR$.



5. In the given figure, in $\triangle PQR$, $XY \parallel QR$, $PX = 1$ cm, $XQ = 3$ cm, $YR = 4.5$ cm and $QR = 9$ cm. Find PY and XY . Further if the area of $\triangle PXY$ is ' A ' cm^2 , find in terms of A the area of $\triangle PQR$ and the area of trapezium $XYRQ$.



6. If A be the area of a right triangle and a one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Aa}{\sqrt{a^4 + 4A^2}}$.

[HOTS]

B. Questions From CBSE Examination Papers

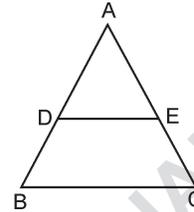
1. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. [2010 (T-I)]
2. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. [2010 (T-I)]
3. State and prove Basic Proportionality theorem. [2010 (T-I)]
4. Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle. [2010 (T-I)]
5. Prove that if a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio. [2010 (T-I)]
6. State and prove converse of Pythagoras theorem. [2010 (T-I)]

7. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Using the above result do the following :

In the figure, $DE \parallel BC$ and $BD = CE$.

Prove that $\triangle ABC$ is an isosceles triangle. [2009]



8. Through the mid-point M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC in L and AD produced in E . Prove that $EL = 2BL$. [2009]

FORMATIVE ASSESSMENT

Activity-1

Objective : To verify the Basic Proportionality Theorem by activity method.

Materials Required : Ruled paper, white sheets of paper, colour pencils, a pair of scissors, geometry box, etc.

Procedure :

1. On a white sheet of paper, draw an acute angled triangle ABC and an obtuse angled triangle PQR . Using a pair of scissors, cut them out.

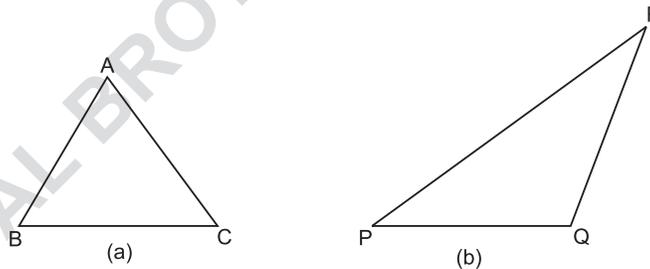


Figure-1

2. Take a ruled paper. Place $\triangle ABC$ over the ruled paper such that any one side of the triangle is placed on one of the lines of the ruled paper. Place the $\triangle PQR$ over the ruled paper in the same manner as discussed above. Mark points X_1, X_2, X_3, X_4 on $\triangle ABC$ and Y_1, Y_2, Y_3, Y_4 on $\triangle PQR$. Join X_1X_2, X_3X_4, Y_1Y_2 , and Y_3Y_4 .

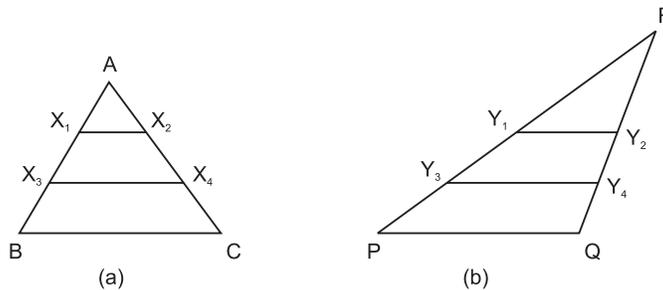


Figure-2

3. Using a ruler, measure $AX_1, X_1B, AX_2, X_2C, AX_3, X_3B, AX_4, X_4C$ and record the lengths in the following table.

AX_1	X_1B	AX_2	X_2C	$\frac{AX_1}{X_1B}$	$\frac{AX_2}{X_2C}$	AX_3	X_3B	AX_4	X_4C	$\frac{AX_3}{X_3B}$	$\frac{AX_4}{X_4C}$

4. Similarly, measure $RY_1, Y_1P, RY_2, Y_2Q, RY_3, Y_3P, RY_4$ and Y_4Q .

RY_1	Y_1P	RY_2	Y_2Q	$\frac{RY_1}{Y_1P}$	$\frac{RY_2}{Y_2Q}$	RY_3	Y_3P	RY_4	Y_4Q	$\frac{RY_3}{Y_3P}$	$\frac{RY_4}{Y_4Q}$

Observations :

1. In figure 2(a), we see that $X_1X_2 \parallel BC$ and $X_3X_4 \parallel BC$
Similarly, in figure 2(b), we see that $Y_1Y_2 \parallel PQ$ and $Y_3Y_4 \parallel PQ$.

2. From the tables, we see that $\frac{AX_1}{X_1B} = \frac{AX_2}{X_2C}, \frac{AX_3}{X_3B} = \frac{AX_4}{X_4C}$

And $\frac{RY_1}{Y_1P} = \frac{RY_2}{Y_2Q}, \frac{RY_3}{Y_3P} = \frac{RY_4}{Y_4Q}$

Thus, we have :

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Conclusion : From the above activity, the Basic Proportionality Theorem is verified.

Do Yourself : Draw two different triangles and verify the Basic Proportionality Theorem by the activity method.

Activity-2

Objective : To verify the Pythagoras Theorem by the method of paper folding, cutting and pasting.

Materials Required : White sheets of paper, tracing paper, a pair of scissors, gluestick, geometry box, etc.

Procedure :

1. On a white sheet of paper, draw a right triangle ABC, right angled at B and $AB = x, BC = y$ and $AC = z$.

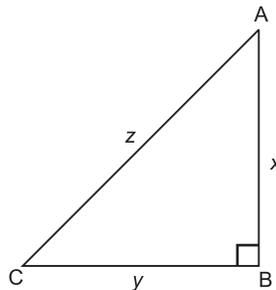


Figure-1

2. Using tracing paper, trace the triangle ABC and make 4 replicas of it. Shade each triangle using different colour. Cut out each triangle.

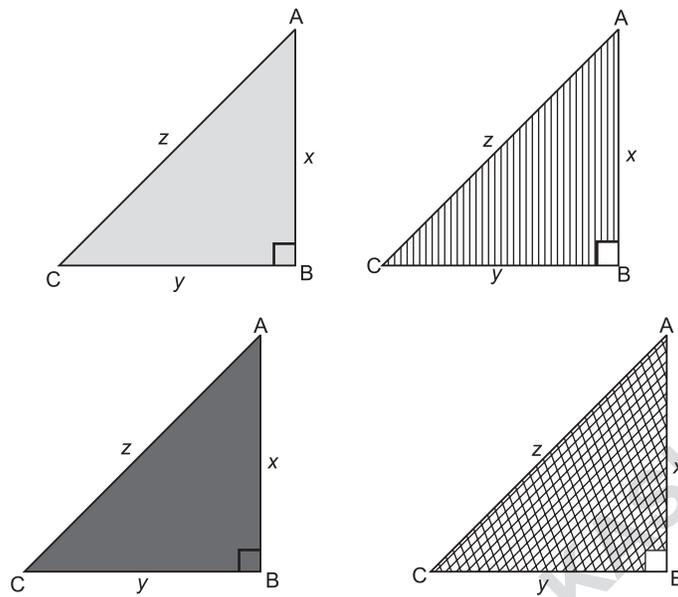


Figure-2

3. On a white sheet of paper, draw a square of side z . Shade it and cut it out.

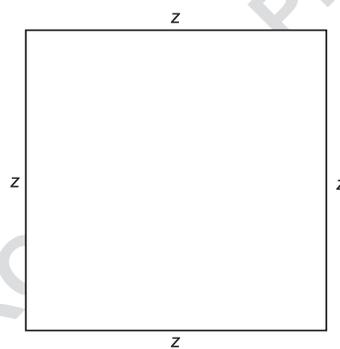


Figure-3

4. Paste the 4 right triangles and the square cut out obtained in steps 2 and 3 above as shown on next page.

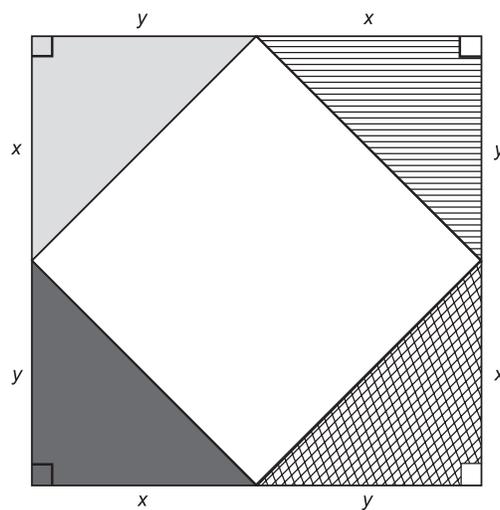


Figure-4

Observations :

1. In the figure 4, we see that it is a square of side $(x + y)$.
So its area = $(x + y)^2$.
2. Also, area of this square = area of the square of side $z + 4 \times$ area of $\triangle ABC$
 $= z^2 + 4 \times \frac{1}{2} \times xy = z^2 + 2xy$
3. From 1 and 2 above, $(x + y)^2 = z^2 + 2xy$
 $\Rightarrow x^2 + y^2 + 2xy = z^2 + 2xy$
 $\Rightarrow x^2 + y^2 = z^2$

Conclusion : From the above activity, it is verified that in a right triangle the square of the hypotenuse is equal to the sum of the squares of other two sides.

Or in other words, we can say that the Pythagoras Theorem is verified.

Do Yourself : Draw two different right triangles and verify the Pythagoras Theorem by Activity method in each case.

Activity-3

Objective : To illustrate that the medians of a triangle concur at a point (called the centroid) which always lies inside the triangle.

Materials Required : White sheets of paper, colour pencils, geometry box, a pair of scissors, gluestick, etc.

Procedure :

1. On a white sheet of paper, draw an acute angled triangle, a right triangle and an obtuse angled triangle. Using a pair of scissors, cut out these triangles.

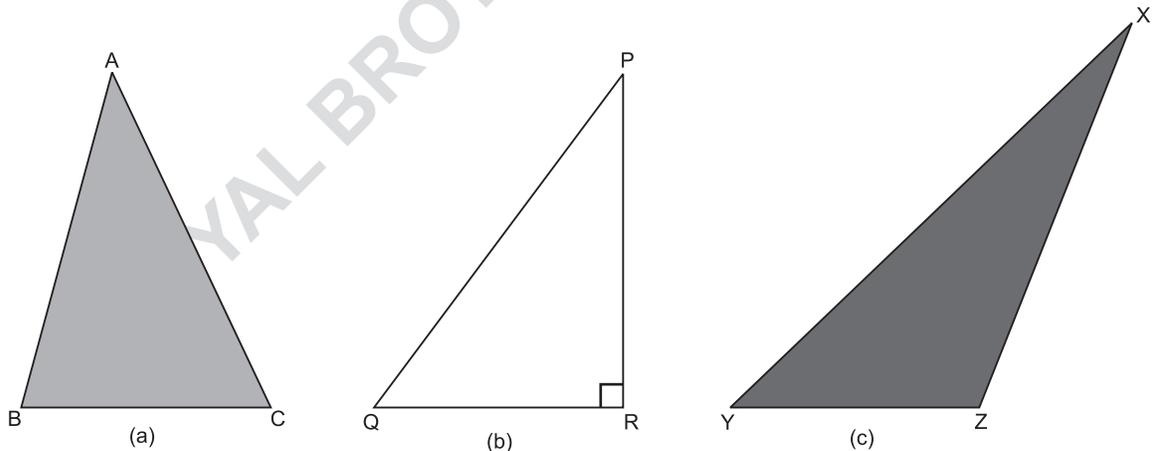


Figure-1

- Take the cut out of the acute angled triangle ABC and find the mid-points of its sides by paper folding method.

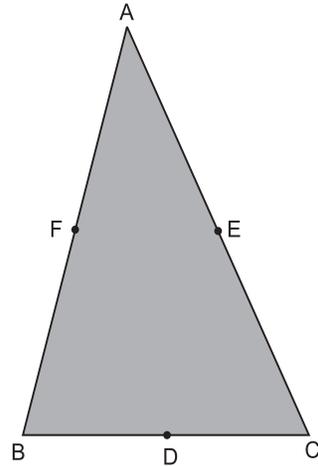


Figure-2

- Let the mid-points of BC, AC and AB be D, E and F respectively.
- Join AD, BE and CF.

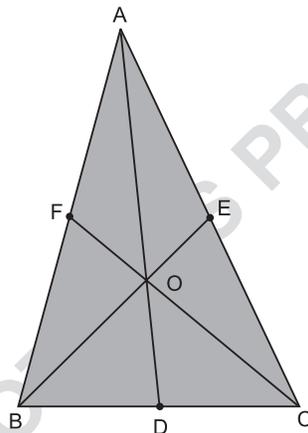


Figure-3

- Repeat the steps 2, 3 and 4 for the right triangle PQR and the obtuse angled triangle XYZ.

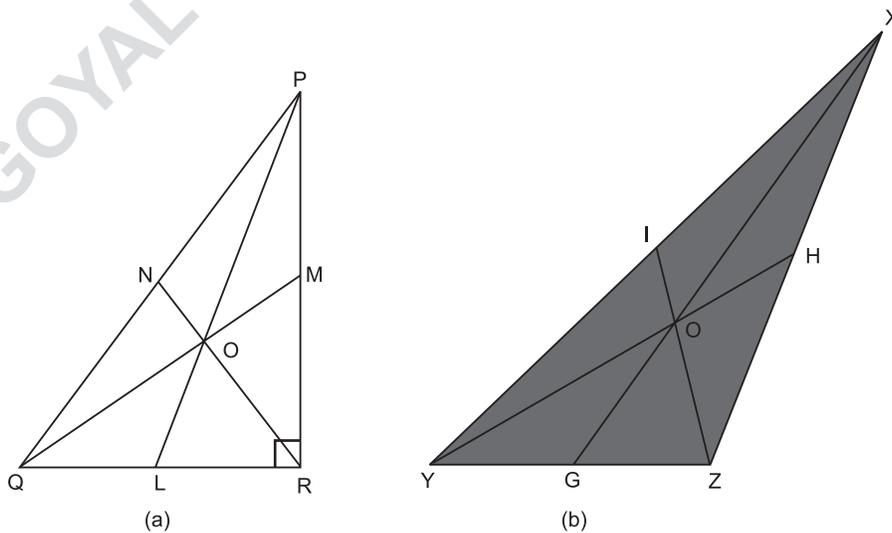


Figure-4

Observations :

1. In figure 3, D, E and F are mid-points of BC, AC and AB respectively. So, AD, BE and CF are medians of ΔABC .
2. Similarly, PL, QM and RN are the medians of ΔPQR [figure 4(a)] and XG, YH and ZI are medians of ΔXYZ [figure 4(b)].
3. We see that in each case the medians pass through a common point, or the medians are concurrent. The point of concurrence of the medians of a triangle is called its centroid.
4. We also observe that in each case the centroid lies in the interior of the triangle.

Conclusion : From the above activity, it is observed that :

- (a) the medians of a triangle are concurrent. This point of concurrence of the medians is called the centroid of the triangle.
- (b) centroid of a triangle always lies in its interior. Or the medians of a triangle always intersect in the interior of the triangle.

Activity-4

Objective : To verify by activity method that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangle on each side of the perpendicular are similar to the whole triangle and to each other.

Materials Required : White sheets of paper, tracing paper, colour pencils, a pair of scissors, gluestick, geometry box, etc.

Procedure :

1. On a white sheet of paper, draw a right triangle ABC, right angled at B.

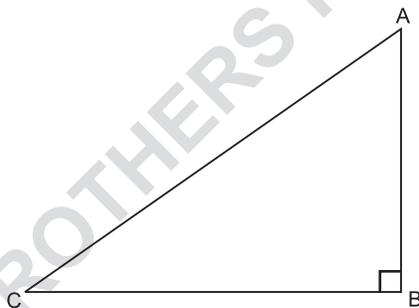


Figure-1

2. On a tracing paper, trace ΔABC . Using paper folding method, draw $BD \perp AC$. Cut the triangles ABD and CBD as shown below. Shade both sides of each triangular cut outs.

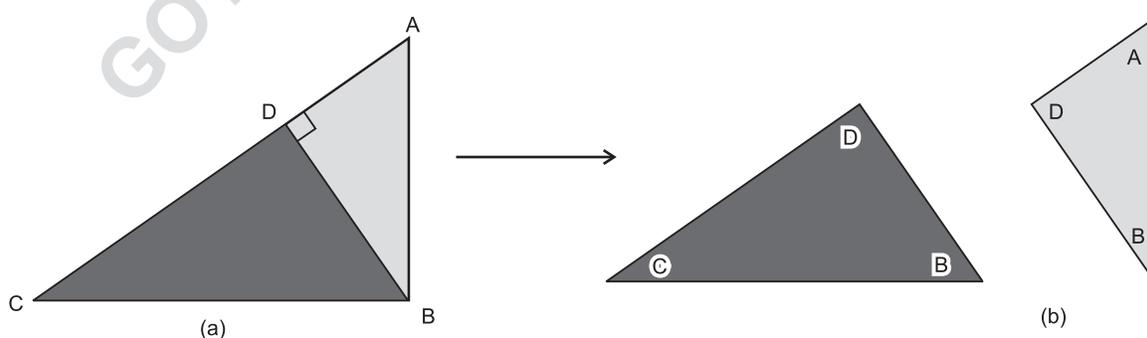


Figure-2

3. Flip over the triangular cut out BCD and place it over ΔABC in figure 1, as shown below.

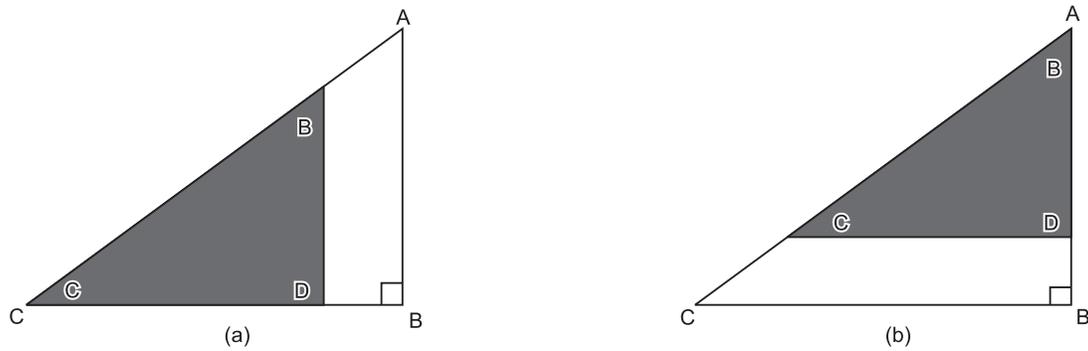


Figure-3

4. Flip over the triangular cut out ABD and place it over ΔABC in figure 1, as shown below.

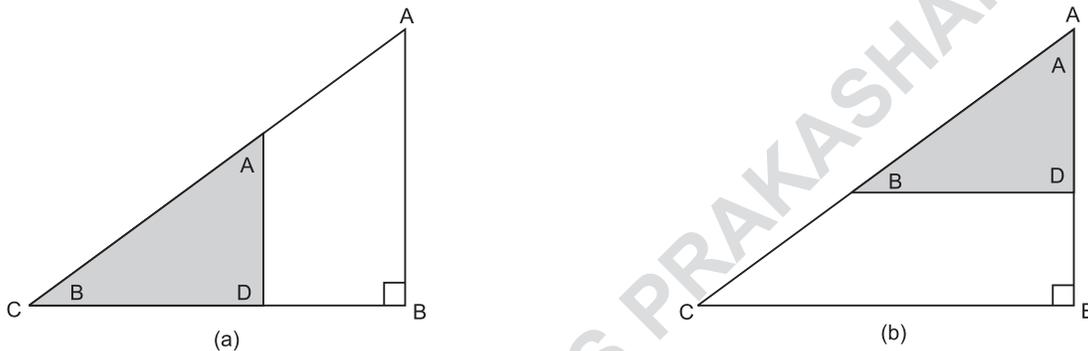


Figure-4

5. Now, take the triangular cut outs BCD and ABD and place them one over another as shown below.



Figure-5

Observations :

1. In figure 2(a), $\angle ABC = 90^\circ$ and in figure 2(b), $\angle CDB = \angle ADB = 90^\circ$.
2. In figure 3(a), and 3(b), we see that $\angle ACB = \angle BCD$ and $\angle BAC = \angle DBC$
So, $\Delta ABC \sim \Delta BDC$
3. In figure 4(a) and 4(b), we see that $\angle ACB = \angle ABD$ and $\angle BAC = \angle DAB$
So, $\Delta ABC \sim \Delta ADB$
4. In figure 5(a) and 5(b), we see that $\angle BCD = \angle ABD$ and $\angle CBD = \angle BAD$
So, $\Delta CBD \sim \Delta BAD$

Conclusion : From the activity, it is verified that if a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

Do Yourself : Draw two different right triangles and verify by activity method that if a perpendicular is drawn from the vertex of the right triangle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

**Exercise 6.1****Question 1:**

Fill in the blanks using correct word given in the brackets: –

- (i) All circles are _____. (congruent, similar)
- (ii) All squares are _____. (similar, congruent)
- (iii) All _____ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

Answer:

- (i) Similar
- (ii) Similar
- (iii) Equilateral
- (iv) (a) Equal
- (b) Proportional

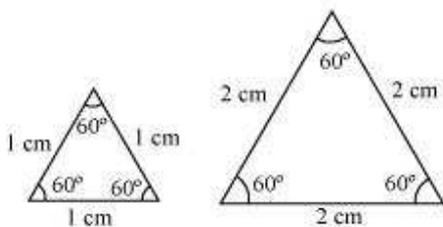
Question 2:

Give two different examples of pair of

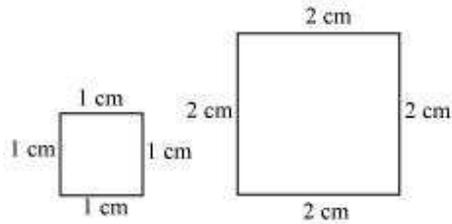
- (i) Similar figures
- (ii) Non-similar figures

Answer:

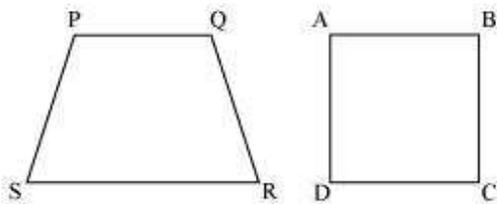
- (i) Two equilateral triangles with sides 1 cm and 2 cm



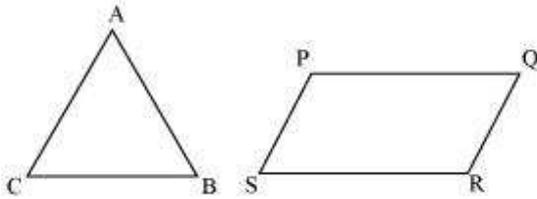
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

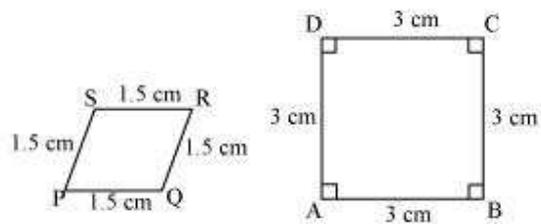


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer:

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

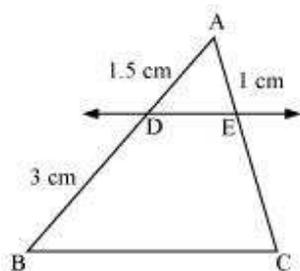


Exercise 6.2

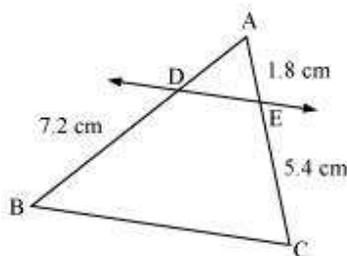
Question 1:

In figure.6.17. (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).

(i)

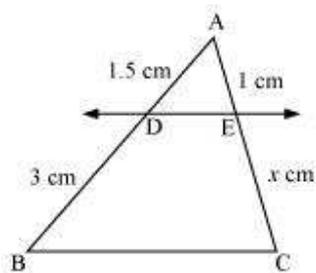


(ii)



Answer:

(i)



Let $EC = x$ cm

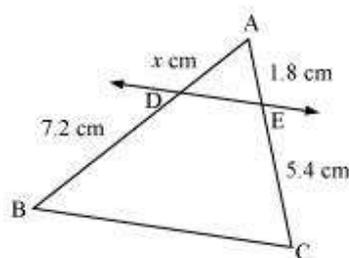
It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain



$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{1.5}{3} = \frac{1}{x}$$
$$x = \frac{3 \times 1}{1.5}$$
$$x = 2$$
$$\therefore EC = 2 \text{ cm}$$

(ii)

Let $AD = x \text{ cm}$ It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{x}{7.2} = \frac{1.8}{5.4}$$
$$x = \frac{1.8 \times 7.2}{5.4}$$
$$x = 2.4$$
$$\therefore AD = 2.4 \text{ cm}$$

Question 2:

E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$.

(i) $PE = 3.9 \text{ cm}$, $EQ = 3 \text{ cm}$, $PF = 3.6 \text{ cm}$ and $FR = 2.4 \text{ cm}$

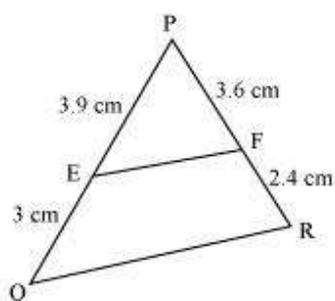


(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.63$ cm

Answer:

(i)



Given that, $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm

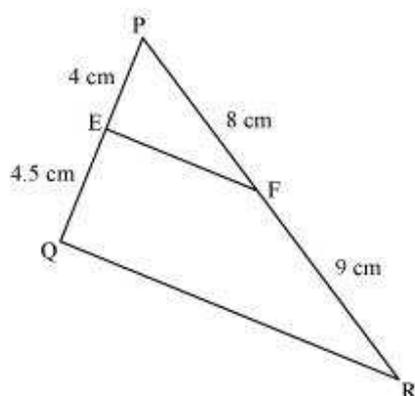
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR .

(ii)



$PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm, $RF = 9$ cm



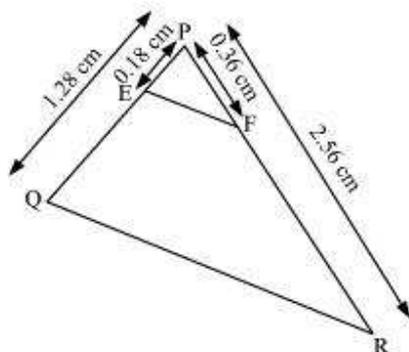
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

$$\text{Hence, } \frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF is parallel to QR.

(iii)



$$PQ = 1.28 \text{ cm, } PR = 2.56 \text{ cm, } PE = 0.18 \text{ cm, } PF = 0.36 \text{ cm}$$

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

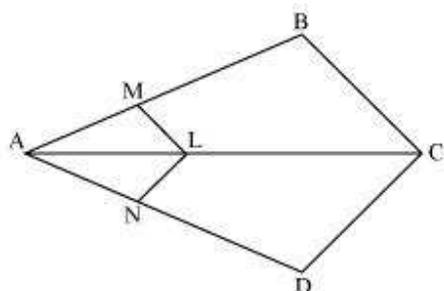
$$\text{Hence, } \frac{PE}{PQ} = \frac{PF}{PR}$$

Therefore, EF is parallel to QR.

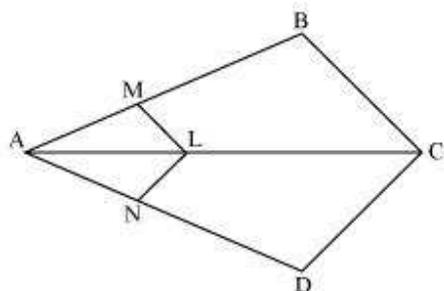
**Question 3:**

In the following figure, if $LM \parallel CB$ and $LN \parallel CD$, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Answer:



In the given figure, $LM \parallel CB$

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \quad (i)$$

Similarly, $LN \parallel CD$

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \quad (ii)$$

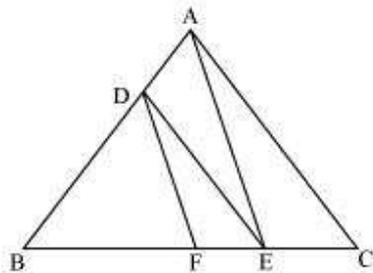
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

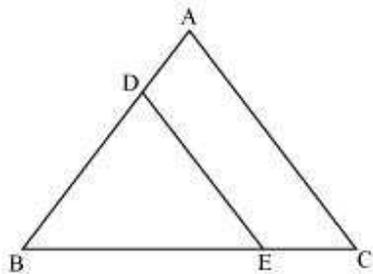
**Question 4:**

In the following figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

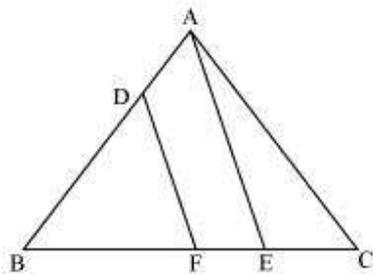


Answer:



In $\triangle ABC$, $DE \parallel AC$

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad (\text{Basic Proportionality Theorem}) \quad (i)$$





In $\triangle BAE$, $DF \parallel AE$

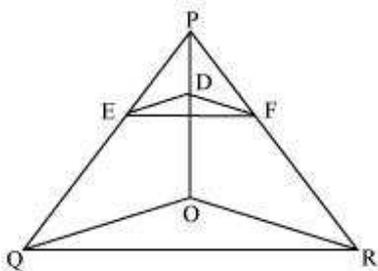
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad (\text{Basic Proportionality Theorem}) \quad (ii)$$

From (i) and (ii), we obtain

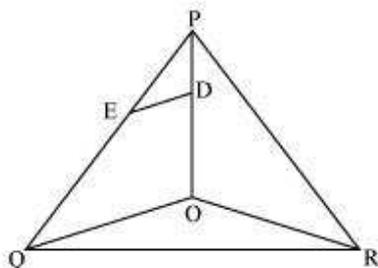
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$.

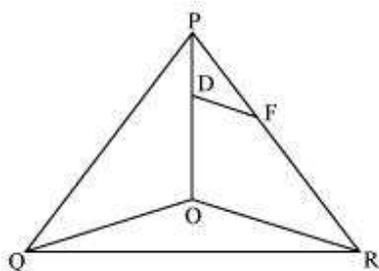


Answer:



In $\triangle POQ$, $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (i)$$



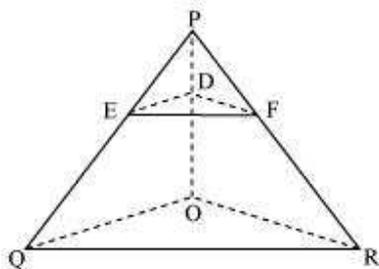
In $\triangle POR$, $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

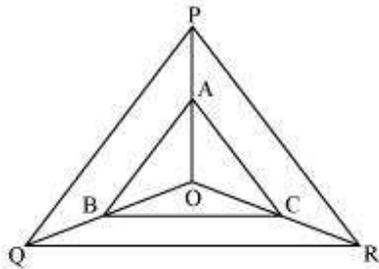
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

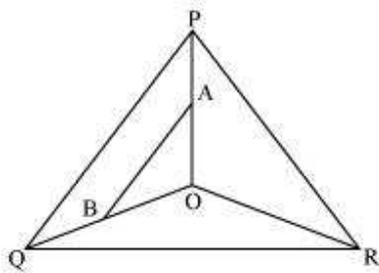


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

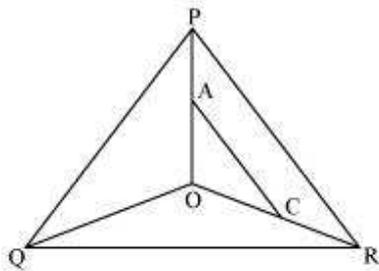


Answer:



In ΔPOQ , $AB \parallel PQ$

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad (\text{Basic proportionality theorem}) \quad (i)$$





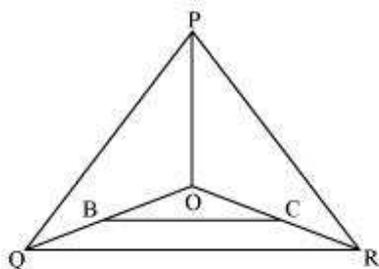
In $\triangle POR$, $AC \parallel PR$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad (\text{By basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

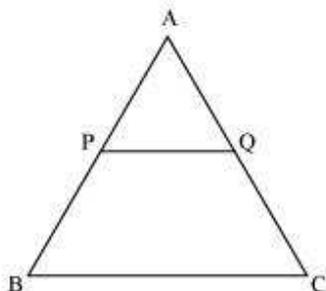
$$\therefore BC \parallel QR \quad (\text{By the converse of basic proportionality theorem})$$



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$



By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \quad (\text{P is the mid-point of AB. } \therefore AP = PB)$$

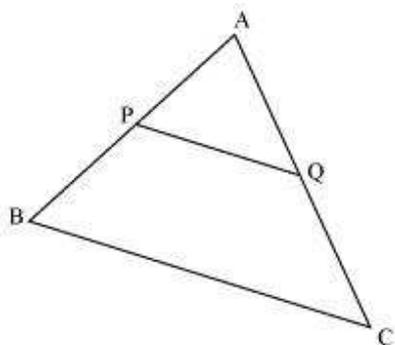
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., $AP = PB$ and $AQ = QC$

It can be observed that



$$\frac{AP}{PB} = \frac{1}{1}$$
$$\text{and } \frac{AQ}{QC} = \frac{1}{1}$$
$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, by using basic proportionality theorem, we obtain

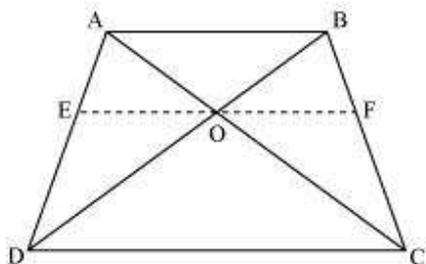
$$PQ \parallel BC$$

Question 9:

ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the

point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer:



Draw a line EF through point O, such that $EF \parallel CD$

In $\triangle ADC$, $EO \parallel CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$$

In $\triangle ABD$, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain



$$\frac{ED}{AE} = \frac{OD}{BO}$$
$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

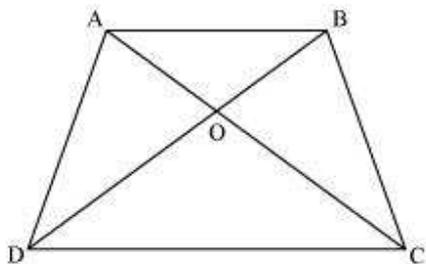
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that

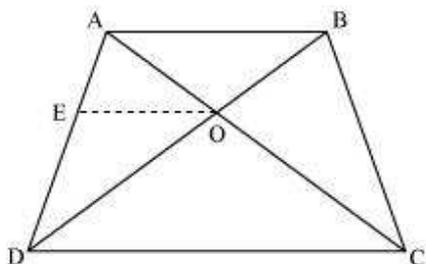
$$\frac{AO}{BO} = \frac{CO}{DO}. \text{ Show that ABCD is a trapezium.}$$

Answer:

Let us consider the following figure for the given question.



Draw a line OE || AB



In $\triangle ABD$, $OE \parallel AB$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

$\Rightarrow EO \parallel DC$ [By the converse of basic proportionality theorem]

$\Rightarrow AB \parallel OE \parallel DC$

$\Rightarrow AB \parallel CD$

$\therefore ABCD$ is a trapezium.

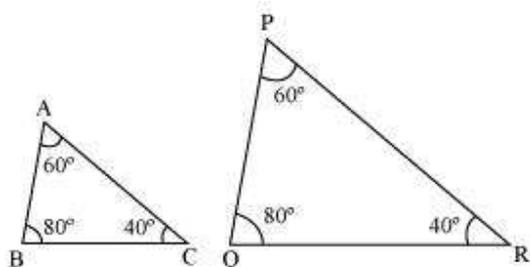


Exercise 6.3

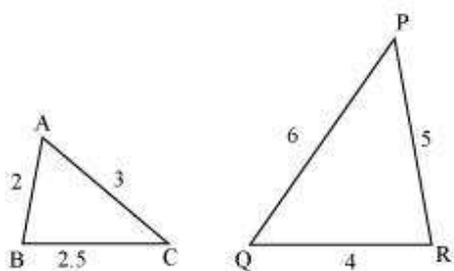
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

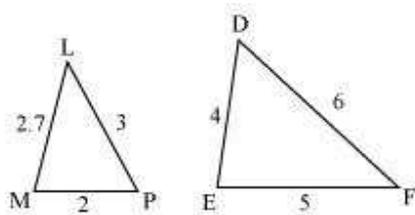
(i)



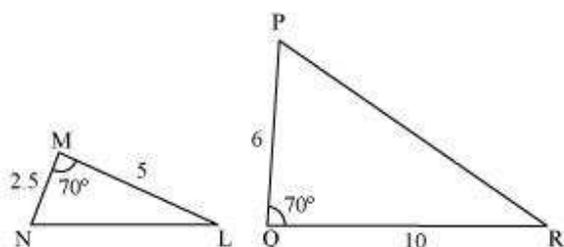
(ii)



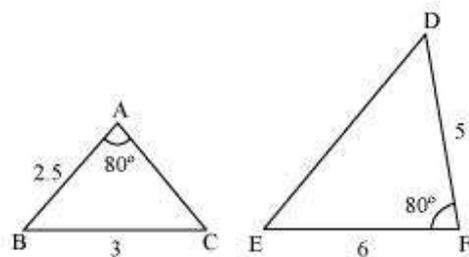
(iii)



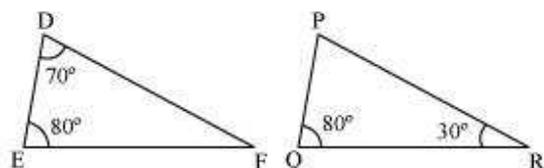
(iv)



(v)



(vi)



Answer:

(i) $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

Therefore, $\triangle ABC \sim \triangle PQR$ [By AAA similarity criterion]

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

(ii)

 $\therefore \triangle ABC \sim \triangle PQR$ [By SSS similarity criterion]

(iii) The given triangles are not similar as the corresponding sides are not proportional.



(iv) The given triangles are not similar as the corresponding sides are not proportional.

(v) The given triangles are not similar as the corresponding sides are not proportional.

(vi) In $\triangle DEF$,

$$\angle D + \angle E + \angle F = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$70^\circ + 80^\circ + \angle F = 180^\circ$$

$$\angle F = 30^\circ$$

Similarly, in $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\angle P + 80^\circ + 30^\circ = 180^\circ$$

$$\angle P = 70^\circ$$

In $\triangle DEF$ and $\triangle PQR$,

$$\angle D = \angle P \text{ (Each } 70^\circ\text{)}$$

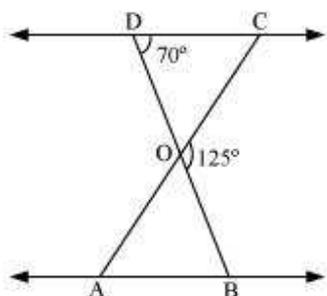
$$\angle E = \angle Q \text{ (Each } 80^\circ\text{)}$$

$$\angle F = \angle R \text{ (Each } 30^\circ\text{)}$$

$\therefore \triangle DEF \sim \triangle PQR$ [By AAA similarity criterion]

Question 2:

In the following figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer:

DOB is a straight line.

$$\therefore \angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ$$

$$= 55^\circ$$

In $\triangle DOC$,

$$\angle DCO + \angle CDO + \angle DOC = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 55^\circ$$

It is given that $\triangle ODC \sim \triangle OBA$.

$$\therefore \angle OAB = \angle OCD \text{ [Corresponding angles are equal in similar triangles.]}$$

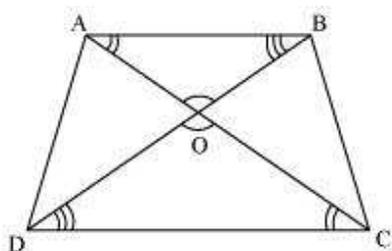
$$\Rightarrow \angle OAB = 55^\circ$$

Question 3:

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the

point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer:



In $\triangle DOC$ and $\triangle BOA$,

$\angle CDO = \angle ABO$ [Alternate interior angles as $AB \parallel CD$]

$\angle DCO = \angle BAO$ [Alternate interior angles as $AB \parallel CD$]

$\angle DOC = \angle BOA$ [Vertically opposite angles]

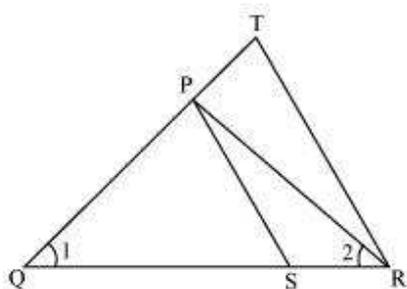
$\therefore \triangle DOC \sim \triangle BOA$ [AAA similarity criterion]

$\therefore \frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

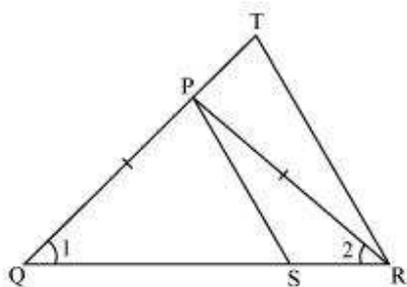
Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$





Answer:



In ΔPQR , $\angle PQR = \angle PRQ$

$\therefore PQ = PR$ (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \quad (ii)$$

In ΔPQS and ΔTQR ,

$$\frac{QR}{QS} = \frac{QT}{QP} \quad [\text{Using (ii)}]$$

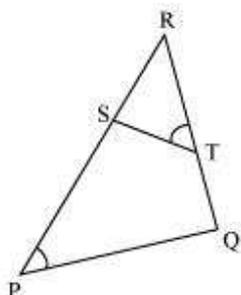
$$\angle Q = \angle Q$$

$\therefore \Delta PQS \sim \Delta TQR$ [SAS similarity criterion]

Question 5:

S and T are point on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Answer:



In $\triangle RPQ$ and $\triangle RST$,

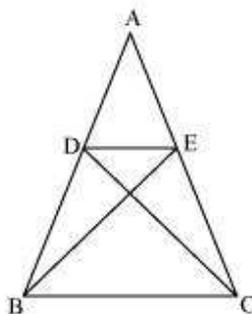
$$\angle RTS = \angle QPS \text{ (Given)}$$

$$\angle R = \angle R \text{ (Common angle)}$$

$\therefore \triangle RPQ \sim \triangle RST$ (By AA similarity criterion)

Question 6:

In the following figure, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.



Answer:

It is given that $\triangle ABE \cong \triangle ACD$.

$$\therefore AB = AC \text{ [By CPCT] (1)}$$

$$\text{And, } AD = AE \text{ [By CPCT] (2)}$$

In $\triangle ADE$ and $\triangle ABC$,

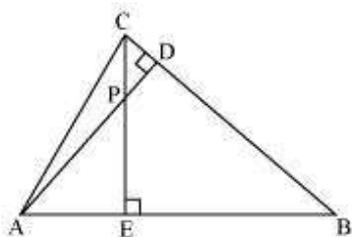
$$\frac{AD}{AB} = \frac{AE}{AC} \text{ [Dividing equation (2) by (1)]}$$

$$\angle A = \angle A \text{ [Common angle]}$$

$\therefore \triangle ADE \sim \triangle ABC$ [By SAS similarity criterion]

**Question 7:**

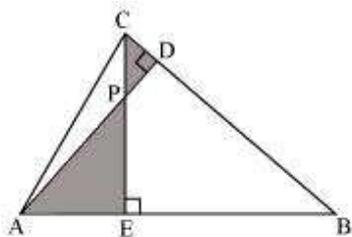
In the following figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:



- (i) $\triangle AEP \sim \triangle CDP$
- (ii) $\triangle ABD \sim \triangle CBE$
- (iii) $\triangle AEP \sim \triangle ADB$
- (v) $\triangle PDC \sim \triangle BEC$

Answer:

(i)



In $\triangle AEP$ and $\triangle CDP$,

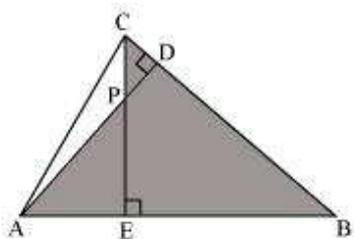
$$\angle AEP = \angle CDP \text{ (Each } 90^\circ\text{)}$$

$$\angle APE = \angle CPD \text{ (Vertically opposite angles)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle CDP$$

(ii)



In $\triangle ABD$ and $\triangle CBE$,

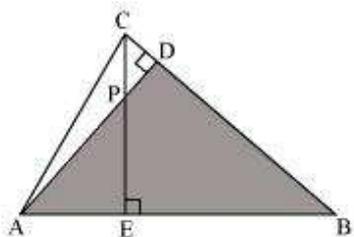
$$\angle ADB = \angle CEB \text{ (Each } 90^\circ\text{)}$$

$$\angle ABD = \angle CBE \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle ABD \sim \triangle CBE$$

(iii)



In $\triangle AEP$ and $\triangle ADB$,

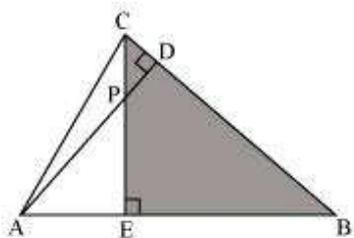
$$\angle AEP = \angle ADB \text{ (Each } 90^\circ\text{)}$$

$$\angle PAE = \angle DAB \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle ADB$$

(iv)



In $\triangle PDC$ and $\triangle BEC$,



$$\angle PDC = \angle BEC \text{ (Each } 90^\circ\text{)}$$

$$\angle PCD = \angle BCE \text{ (Common angle)}$$

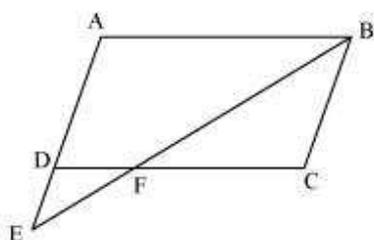
Hence, by using AA similarity criterion,

$$\triangle PDC \sim \triangle BEC$$

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$

Answer:



In $\triangle ABE$ and $\triangle CFB$,

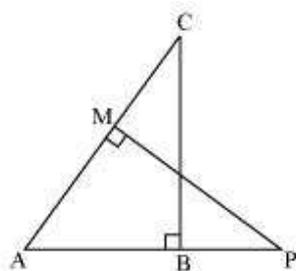
$$\angle A = \angle C \text{ (Opposite angles of a parallelogram)}$$

$$\angle AEB = \angle CBF \text{ (Alternate interior angles as } AE \parallel BC\text{)}$$

$$\therefore \triangle ABE \sim \triangle CFB \text{ (By AA similarity criterion)}$$

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\triangle ABC \sim \triangle AMP$



$$(ii) \frac{CA}{PA} = \frac{BC}{MP}$$

Answer:

In $\triangle ABC$ and $\triangle AMP$,

$$\angle ABC = \angle AMP \text{ (Each } 90^\circ\text{)}$$

$$\angle A = \angle A \text{ (Common)}$$

$\therefore \triangle ABC \sim \triangle AMP$ (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \quad \text{(Corresponding sides of similar triangles are proportional)}$$

Question 10:

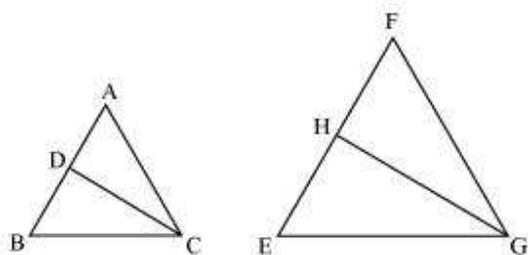
CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle FEG$ respectively. If $\triangle ABC \sim \triangle FEG$, Show that:

$$(i) \frac{CD}{GH} = \frac{AC}{FG}$$

$$(ii) \triangle DCB \sim \triangle HGE$$

$$(iii) \triangle DCA \sim \triangle HGF$$

Answer:



It is given that $\triangle ABC \sim \triangle FEG$.

$$\therefore \angle A = \angle F, \angle B = \angle E, \text{ and } \angle ACB = \angle FGE$$

$$\angle ACB = \angle FGE$$

$$\therefore \angle ACD = \angle FGH \text{ (Angle bisector)}$$

$$\text{And, } \angle DCB = \angle HGE \text{ (Angle bisector)}$$

In $\triangle ACD$ and $\triangle FGH$,



$\angle A = \angle F$ (Proved above)

$\angle ACD = \angle FGH$ (Proved above)

$\therefore \triangle ACD \sim \triangle FGH$ (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In $\triangle DCB$ and $\triangle HGE$,

$\angle DCB = \angle HGE$ (Proved above)

$\angle B = \angle E$ (Proved above)

$\therefore \triangle DCB \sim \triangle HGE$ (By AA similarity criterion)

In $\triangle DCA$ and $\triangle HGF$,

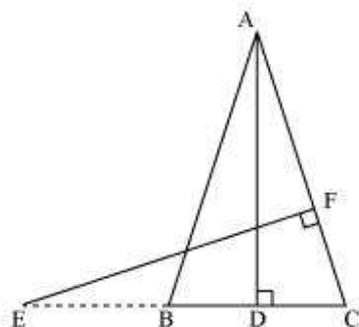
$\angle ACD = \angle FGH$ (Proved above)

$\angle A = \angle F$ (Proved above)

$\therefore \triangle DCA \sim \triangle HGF$ (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$



Answer:

It is given that ABC is an isosceles triangle.

$\therefore AB = AC$

$\Rightarrow \angle ABD = \angle ECF$

In $\triangle ABD$ and $\triangle ECF$,



$$\angle ADB = \angle EFC \text{ (Each } 90^\circ)$$

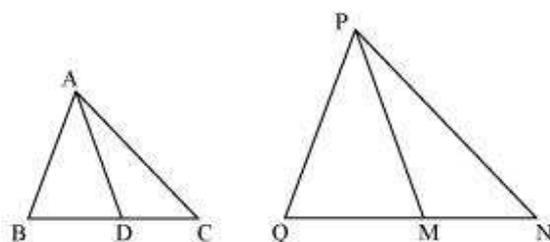
$$\angle BAD = \angle CEF \text{ (Proved above)}$$

$$\therefore \triangle ABD \sim \triangle ECF \text{ (By using AA similarity criterion)}$$

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see the given figure). Show that $\triangle ABC \sim \triangle PQR$.

Answer:



Median divides the opposite side.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2}$$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In $\triangle ABD$ and $\triangle PQM$,



$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \text{ (Proved above)}$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SSS similarity criterion)

$\Rightarrow \angle ABD = \angle PQM$ (Corresponding angles of similar triangles)

In $\triangle ABC$ and $\triangle PQR$,

$\angle ABD = \angle PQM$ (Proved above)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

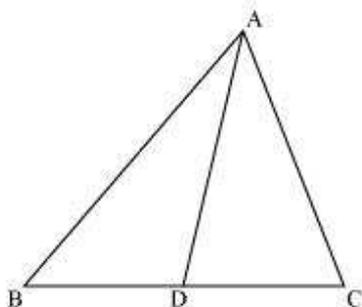
$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that

$$CA^2 = CB \cdot CD.$$

Answer:



In $\triangle ADC$ and $\triangle BAC$,

$\angle ADC = \angle BAC$ (Given)

$\angle ACD = \angle BCA$ (Common angle)

$\therefore \triangle ADC \sim \triangle BAC$ (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

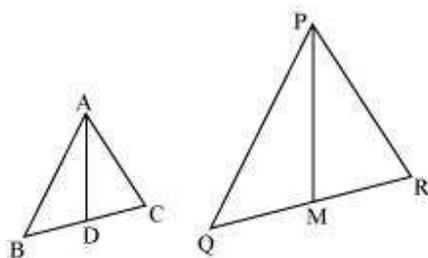
$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

**Question 14:**

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

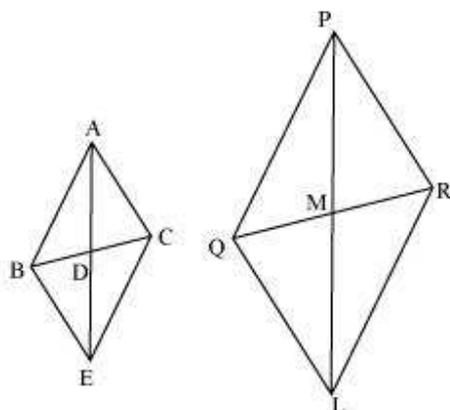
Answer:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)



In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

$\therefore AC = BE$ and $AB = EC$ (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and $PR = QL$, $PQ = LR$

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

$\therefore \triangle ABE \sim \triangle PQL$ (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

$$\therefore \angle BAE = \angle QPL \dots (1)$$

Similarly, it can be proved that $\triangle AEC \sim \triangle PLR$ and

$$\angle CAE = \angle RPL \dots (2)$$

Adding equation (1) and (2), we obtain

$$\angle BAE + \angle CAE = \angle QPL + \angle RPL$$

$$\Rightarrow \angle CAB = \angle RPQ \dots (3)$$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ (Given)}$$

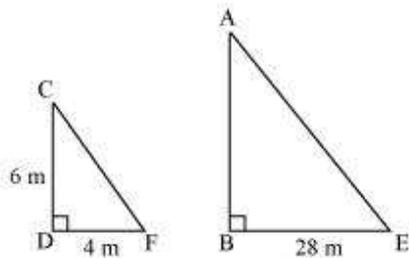
$$\angle CAB = \angle RPQ \text{ [Using equation (3)]}$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

**Question 15:**

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \triangle ABE \sim \triangle CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

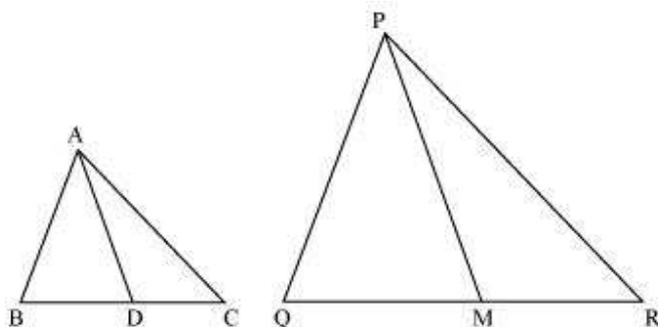
Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where

$$\triangle ABC \sim \triangle PQR \text{ prove that } \frac{AB}{PQ} = \frac{AD}{PM}$$

Answer:



It is given that $\Delta ABC \sim \Delta PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots (1)$$

$$\text{Also, } \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$$

Since AD and PM are medians, they will divide their opposite sides.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \dots (3)$$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots (4)$$

In ΔABD and ΔPQM ,

$$\angle B = \angle Q \text{ [Using equation (2)]}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ [Using equation (4)]}$$

$\therefore \Delta ABD \sim \Delta PQM$ (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

**Exercise 6.4****Question 1:**

Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2 . If $EF = 15.4 \text{ cm}$, find BC .

Answer:

It is given that $\triangle ABC \sim \triangle DEF$.

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$$EF = 15.4 \text{ cm},$$

$$\text{ar}(\triangle ABC) = 64 \text{ cm}^2,$$

$$\text{ar}(\triangle DEF) = 121 \text{ cm}^2$$

$$\therefore \frac{\text{ar}(ABC)}{\text{ar}(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64 \text{ cm}^2}{121 \text{ cm}^2}\right) = \frac{BC^2}{(15.4 \text{ cm})^2}$$

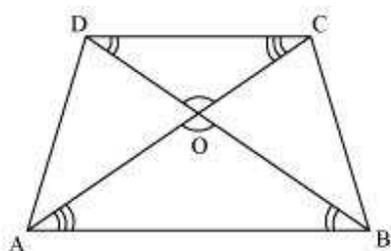
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right) \text{ cm}$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right) \text{ cm} = (8 \times 1.4) \text{ cm} = 11.2 \text{ cm}$$

Question 2:

Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O . If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD .

Answer:



Since $AB \parallel CD$,

$\therefore \angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$ (Alternate interior angles)

In $\triangle AOB$ and $\triangle COD$,

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle OBA = \angle ODC$ (Alternate interior angles)

$\therefore \triangle AOB \sim \triangle COD$ (By AAA similarity criterion)

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{AB}{CD}\right)^2$$

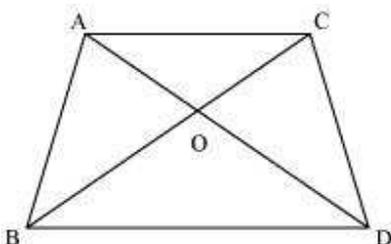
Since $AB = 2 CD$,

$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{2 CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC . If AD

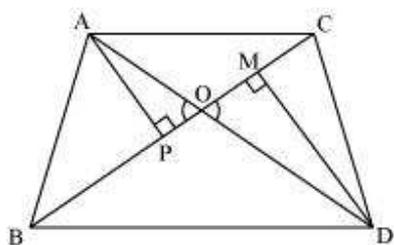
intersects BC at O , show that $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$





Answer:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

In $\triangle APO$ and $\triangle DMO$,

$\angle APO = \angle DMO$ (Each = 90°)

$\angle AOP = \angle DOM$ (Vertically opposite angles)

$\therefore \triangle APO \sim \triangle DMO$ (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer:

Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$.



$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \quad (1)$$

Given that, $\text{ar}(\Delta ABC) = \text{ar}(\Delta PQR)$

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

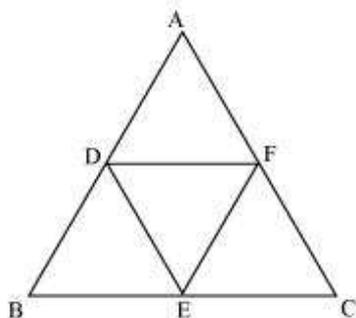
$\Rightarrow AB = PQ, BC = QR, \text{ and } AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC . Find the ratio of the area of ΔDEF and ΔABC .

Answer:



D and E are the mid-points of ΔABC .



$$\therefore DE \parallel AC \text{ and } DE = \frac{1}{2} AC$$

In $\triangle BED$ and $\triangle BCA$,

$$\angle BED = \angle BCA \quad (\text{Corresponding angles})$$

$$\angle BDE = \angle BAC \quad (\text{Corresponding angles})$$

$$\angle EBD = \angle CBA \quad (\text{Common angles})$$

$$\therefore \triangle BED \sim \triangle BCA \quad (\text{AAA similarity criterion})$$

$$\frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \frac{1}{4}$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle BCA)$$

$$\text{Similarly, } \text{ar}(\triangle CFE) = \frac{1}{4} \text{ar}(\triangle CBA) \text{ and } \text{ar}(\triangle ADF) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\text{Also, } \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - [\text{ar}(\triangle BED) + \text{ar}(\triangle CFE) + \text{ar}(\triangle ADF)]$$

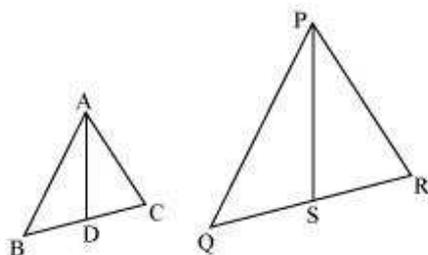
$$\Rightarrow \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - \frac{3}{4} \text{ar}(\triangle ABC) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\Rightarrow \frac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer:



Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$. Let AD and PS be the medians of these triangles.

$$\because \Delta ABC \sim \Delta PQR$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \dots(1)$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$$

Since AD and PS are medians,

$$\therefore BD = DC = \frac{BC}{2}$$

$$\text{And, } QS = SR = \frac{QR}{2}$$

Equation (1) becomes

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR} \dots (3)$$

In ΔABD and ΔPQS ,

$$\angle B = \angle Q \text{ [Using equation (2)]}$$

$$\text{And, } \frac{AB}{PQ} = \frac{BD}{QS} \text{ [Using equation (3)]}$$

$$\therefore \Delta ABD \sim \Delta PQS \text{ (SAS similarity criterion)}$$

Therefore, it can be said that

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS} \dots (4)$$



$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

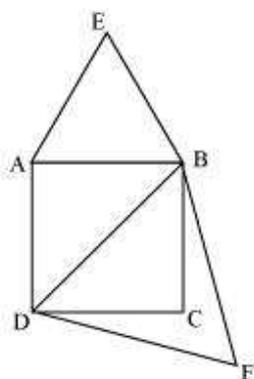
And hence,

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer:



Let ABCD be a square of side a .

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF .

Side of an equilateral triangle, ΔABE , described on one of its sides $= a$

Side of an equilateral triangle, ΔDBF , described on one of its diagonals $= \sqrt{2}a$



We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

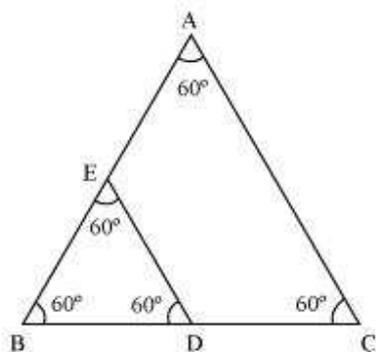
$$\frac{\text{Area of } \triangle ABE}{\text{Area of } \triangle DBF} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

- (A) 2 : 1
- (B) 1 : 2
- (C) 4 : 1
- (D) 1 : 4

Answer:



We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of $\triangle ABC = x$

Therefore, side of $\triangle BDE = \frac{x}{2}$



$$\therefore \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

- (A) 2 : 3
- (B) 4 : 9
- (C) 81 : 16
- (D) 16 : 81

Answer:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

**Exercise 6.5****Question 1:**

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer:

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

$$49 + 576 = 625$$

$$\text{Or, } 7^2 + 24^2 = 25^2$$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

$$\text{However, } 9 + 36 \neq 64$$

$$\text{Or, } 3^2 + 6^2 \neq 8^2$$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii) Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

$$\text{However, } 2500 + 6400 \neq 10000$$

$$\text{Or, } 50^2 + 80^2 \neq 100^2$$



Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv) Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, $144 + 25 = 169$

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

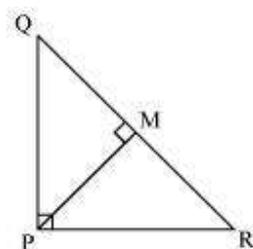
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \times MR$.

Answer:





Let $\angle MPR = x$

In $\triangle MPR$,

$$\angle MRP = 180^\circ - 90^\circ - x$$

$$\angle MRP = 90^\circ - x$$

Similarly, in $\triangle MPQ$,

$$\angle MPQ = 90^\circ - \angle MPR$$

$$= 90^\circ - x$$

$$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$$

$$\angle MQP = x$$

In $\triangle QMP$ and $\triangle PMR$,

$$\angle MPQ = \angle MRP$$

$$\angle PMQ = \angle RMP$$

$$\angle MQP = \angle MPR$$

$\therefore \triangle QMP \sim \triangle PMR$ (By AAA similarity criterion)

$$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$$

$$\Rightarrow PM^2 = QM \times MR$$

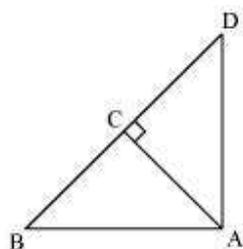
Question 3:

In the following figure, $\triangle ABD$ is a triangle right angled at A and $AC \perp BD$. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$



Answer:

(i) In $\triangle ADB$ and $\triangle CAB$,



$$\angle DAB = \angle ACB \quad (\text{Each } 90^\circ)$$

$$\angle ABD = \angle CBA \quad (\text{Common angle})$$

$$\therefore \triangle ADB \sim \triangle CAB \quad (\text{AA similarity criterion})$$

$$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$$

$$\Rightarrow AB^2 = CB \times BD$$

(ii) Let $\angle CAB = x$

In $\triangle CBA$,

$$\angle CBA = 180^\circ - 90^\circ - x$$

$$\angle CBA = 90^\circ - x$$

Similarly, in $\triangle CAD$,

$$\angle CAD = 90^\circ - \angle CAB$$

$$= 90^\circ - x$$

$$\angle CDA = 180^\circ - 90^\circ - (90^\circ - x)$$

$$\angle CDA = x$$

In $\triangle CBA$ and $\triangle CAD$,

$$\angle CBA = \angle CAD$$

$$\angle CAB = \angle CDA$$

$$\angle ACB = \angle DCA \quad (\text{Each } 90^\circ)$$

$$\therefore \triangle CBA \sim \triangle CAD \quad (\text{By AAA rule})$$

$$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$$

$$\Rightarrow AC^2 = DC \times BC$$

(iii) In $\triangle DCA$ and $\triangle DAB$,

$$\angle DCA = \angle DAB \quad (\text{Each } 90^\circ)$$

$$\angle CDA = \angle ADB \quad (\text{Common angle})$$

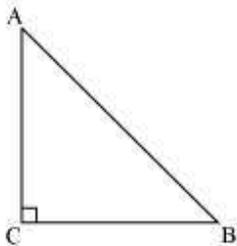


$$\begin{aligned} \therefore \Delta DCA &\sim \Delta DAB && \text{(AA similarity criterion)} \\ \Rightarrow \frac{DC}{DA} &= \frac{DA}{DB} \\ \Rightarrow AD^2 &= BD \times CD \end{aligned}$$

Question 4:

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2 AC^2$.

Answer:



Given that ΔABC is an isosceles triangle.

$$\therefore AC = CB$$

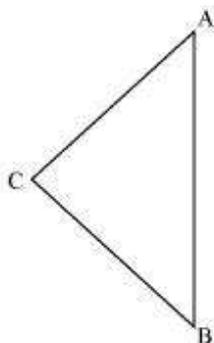
Applying Pythagoras theorem in ΔABC (i.e., right-angled at point C), we obtain

$$\begin{aligned} AC^2 + CB^2 &= AB^2 \\ \Rightarrow AC^2 + AC^2 &= AB^2 && (AC=CB) \\ \Rightarrow 2AC^2 &= AB^2 \end{aligned}$$

Question 5:

ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2 AC^2$, prove that ABC is a right triangle.

Answer:



Given that,

$$AB^2 = 2AC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$$

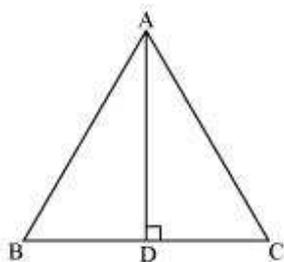
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Question 6:

ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Answer:



Let AD be the altitude in the given equilateral triangle, ΔABC .

We know that altitude bisects the opposite side.

$$\therefore BD = DC = a$$



In $\triangle ADB$,

$$\angle ADB = 90^\circ$$

Applying pythagoras theorem, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 + a^2 = (2a)^2$$

$$\Rightarrow AD^2 + a^2 = 4a^2$$

$$\Rightarrow AD^2 = 3a^2$$

$$\Rightarrow AD = a\sqrt{3}$$

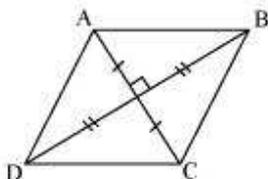
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Question 7:

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer:



In $\triangle AOB$, $\triangle BOC$, $\triangle COD$, $\triangle AOD$,

Applying Pythagoras theorem, we obtain



$$AB^2 = AO^2 + OB^2 \quad \dots (1)$$

$$BC^2 = BO^2 + OC^2 \quad \dots (2)$$

$$CD^2 = CO^2 + OD^2 \quad \dots (3)$$

$$AD^2 = AO^2 + OD^2 \quad \dots (4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + OB^2 + OC^2 + OD^2)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

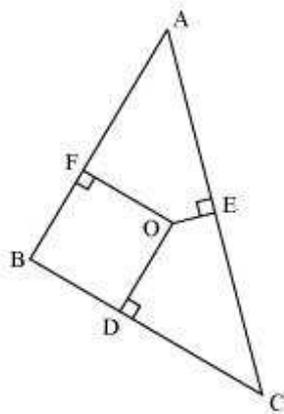
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Question 8:

In the following figure, O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

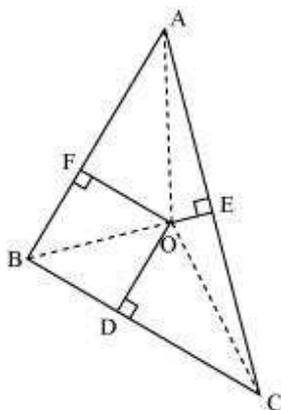


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer:

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in $\triangle AOF$, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in $\triangle BOD$,

$$OB^2 = OD^2 + BD^2$$

Similarly, in $\triangle COE$,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

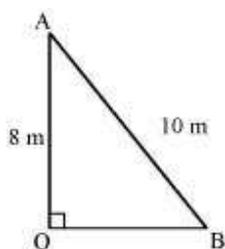
$$AF^2 + BD^2 + EC^2 = (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer:



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$

$$(10 \text{ m})^2 = (8 \text{ m})^2 + OB^2$$

$$100 \text{ m}^2 = 64 \text{ m}^2 + OB^2$$

$$OB^2 = 36 \text{ m}^2$$

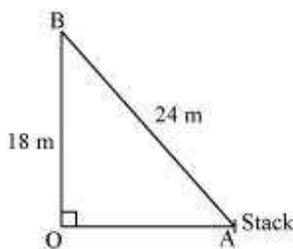
$$OB = 6 \text{ m}$$

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer:



Let OB be the pole and AB be the wire.

By Pythagoras theorem,



$$AB^2 = OB^2 + OA^2$$

$$(24 \text{ m})^2 = (18 \text{ m})^2 + OA^2$$

$$OA^2 = (576 - 324) \text{ m}^2 = 252 \text{ m}^2$$

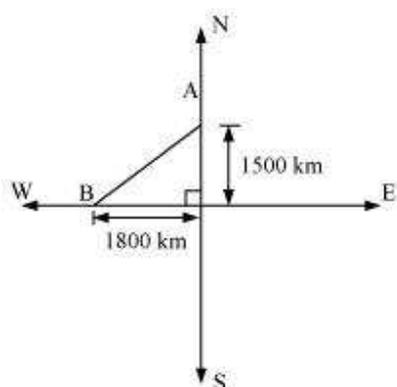
$$OA = \sqrt{252} \text{ m} = \sqrt{6 \times 6 \times 7} \text{ m} = 6\sqrt{7} \text{ m}$$

Therefore, the distance from the base is $6\sqrt{7}$ m.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer:



Distance travelled by the plane flying towards north in $1\frac{1}{2}$ hrs
 $= 1,000 \times 1\frac{1}{2} = 1,500 \text{ km}$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}$ hrs
 $= 1,200 \times 1\frac{1}{2} = 1,800 \text{ km}$



Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

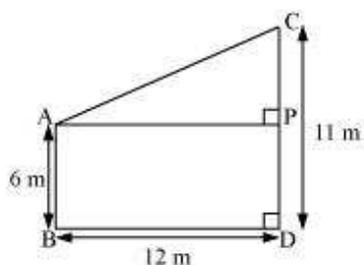
$$\begin{aligned} \text{Distance between these planes after } 1\frac{1}{2} \text{ hrs, } AB &= \sqrt{OA^2 + OB^2} \\ &= \left(\sqrt{(1,500)^2 + (1,800)^2} \right) \text{ km} = \left(\sqrt{2250000 + 3240000} \right) \text{ km} \\ &= \left(\sqrt{5490000} \right) \text{ km} = \left(\sqrt{9 \times 610000} \right) \text{ km} = 300\sqrt{61} \text{ km} \end{aligned}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}$ hrs.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer:



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, $CP = 11 - 6 = 5$ m

From the figure, it can be observed that $AP = 12$ m

Applying Pythagoras theorem for ΔAPC , we obtain



$$AP^2 + PC^2 = AC^2$$

$$(12 \text{ m})^2 + (5 \text{ m})^2 = AC^2$$

$$AC^2 = (144 + 25)m^2 = 169 \text{ m}^2$$

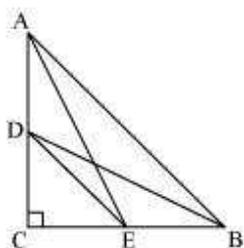
$$AC = 13 \text{ m}$$

Therefore, the distance between their tops is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Answer:



Applying Pythagoras theorem in $\triangle ACE$, we obtain

$$AC^2 + CE^2 = AE^2 \quad \dots (1)$$

Applying Pythagoras theorem in $\triangle BCD$, we obtain

$$BC^2 + CD^2 = BD^2 \quad \dots (2)$$

Using equation (1) and equation (2), we obtain

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \dots (3)$$

Applying Pythagoras theorem in $\triangle CDE$, we obtain

$$DE^2 = CD^2 + CE^2$$

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AB^2 = AC^2 + CB^2$$

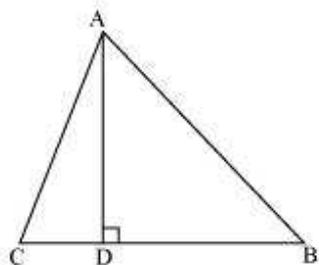
Putting the values in equation (3), we obtain

$$DE^2 + AB^2 = AE^2 + BD^2$$

Question 14:



The perpendicular from A on side BC of a ΔABC intersect BC at D such that $DB = 3 CD$. Prove that $2 AB^2 = 2 AC^2 + BC^2$



Answer:

Applying Pythagoras theorem for ΔACD , we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \quad \dots (1)$$

Applying Pythagoras theorem in ΔABD , we obtain

$$AB^2 = AD^2 + DB^2$$

$$AD^2 = AB^2 - DB^2 \quad \dots (2)$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \quad \dots (3)$$

It is given that $3DC = DB$

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

$$16AB^2 - 16AC^2 = 8BC^2$$

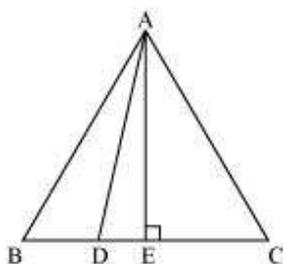
$$2AB^2 = 2AC^2 + BC^2$$

Question 15:



In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3} BC$. Prove that $9 AD^2 = 7 AB^2$.

Answer:



Let the side of the equilateral triangle be a , and AE be the altitude of $\triangle ABC$.

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

$$\text{And, } AE = \frac{a\sqrt{3}}{2}$$

$$\text{Given that, } BD = \frac{1}{3} BC$$

$$\therefore BD = \frac{a}{3}$$

$$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

Applying Pythagoras theorem in $\triangle ADE$, we obtain

$$AD^2 = AE^2 + DE^2$$



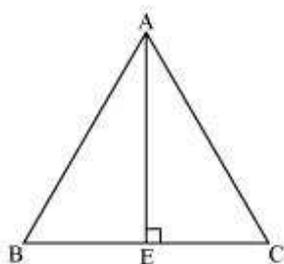
$$\begin{aligned}AD^2 &= \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2 \\&= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right) \\&= \frac{28a^2}{36} \\&= \frac{7}{9}AB^2\end{aligned}$$

$$\Rightarrow 9 AD^2 = 7 AB^2$$

Question 16:

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer:



Let the side of the equilateral triangle be a , and AE be the altitude of $\triangle ABC$.

$$\therefore BE = EC = \frac{BC}{2} = \frac{a}{2}$$

Applying Pythagoras theorem in $\triangle ABE$, we obtain

$$AB^2 = AE^2 + BE^2$$



$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$$4AE^2 = 3a^2$$

$$\Rightarrow 4 \times (\text{Square of altitude}) = 3 \times (\text{Square of one side})$$

Question 17:

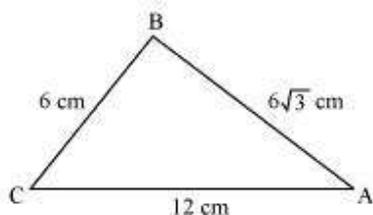
Tick the correct answer and justify: In $\triangle ABC$, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer:



Given that, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm, and $BC = 6$ cm

It can be observed that

$$AB^2 = 108$$

$$AC^2 = 144$$

$$\text{And, } BC^2 = 36$$

$$AB^2 + BC^2 = AC^2$$

The given triangle, $\triangle ABC$, is satisfying Pythagoras theorem.



Therefore, the triangle is a right triangle, right-angled at B.

$$\therefore \angle B = 90^\circ$$

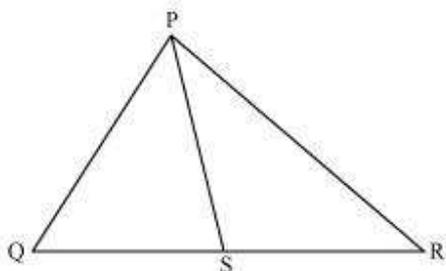
Hence, the correct answer is (C).



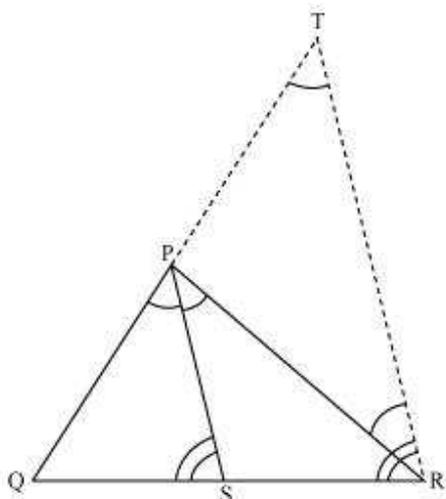
Exercise 6.6

Question 1:

In the given figure, PS is the bisector of $\angle QPR$ of ΔPQR . Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Answer:



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of $\angle QPR$.

$$\angle QPS = \angle SPR \dots (1)$$

By construction,

$$\angle SPR = \angle PRT \text{ (As } PS \parallel TR \text{)} \dots (2)$$

$$\angle QPS = \angle QTR \text{ (As } PS \parallel TR \text{)} \dots (3)$$



Using these equations, we obtain

$$\angle PRT = \angle QTR$$

$$\therefore PT = PR$$

By construction,

$$PS \parallel TR$$

By using basic proportionality theorem for ΔQTR ,

$$\frac{QS}{SR} = \frac{QP}{PT}$$

$$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR} \quad (PT = TR)$$

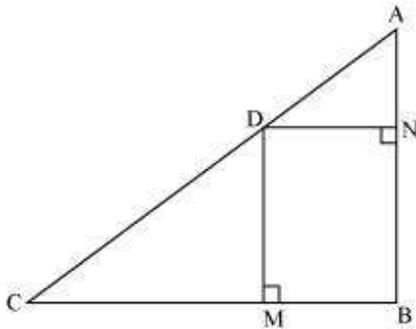
Question 2:

In the given figure, D is a point on hypotenuse AC of ΔABC , $DM \perp BC$ and $DN \perp AB$,

Prove that:

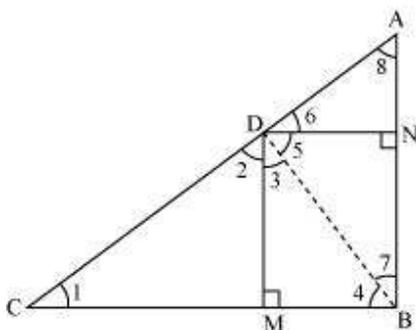
(i) $DM^2 = DN \cdot MC$

(ii) $DN^2 = DM \cdot AN$



Answer:

(i) Let us join DB.



We have, $DN \parallel CB$, $DM \parallel AB$, and $\angle B = 90^\circ$

\therefore DMBN is a rectangle.

$\therefore DN = MB$ and $DM = NB$

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle CDB = 90^\circ$

$\Rightarrow \angle 2 + \angle 3 = 90^\circ \dots (1)$

In $\triangle CDM$,

$\angle 1 + \angle 2 + \angle DMC = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ \dots (2)$

In $\triangle DMB$,

$\angle 3 + \angle DMB + \angle 4 = 180^\circ$

$\Rightarrow \angle 3 + \angle 4 = 90^\circ \dots (3)$

From equation (1) and (2), we obtain

$\angle 1 = \angle 3$

From equation (1) and (3), we obtain

$\angle 2 = \angle 4$

In $\triangle DCM$ and $\triangle BDM$,

$\angle 1 = \angle 3$ (Proved above)

$\angle 2 = \angle 4$ (Proved above)

$\therefore \triangle DCM \sim \triangle BDM$ (AA similarity criterion)



$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$
$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \quad (BM = DN)$$

$$\Rightarrow DM^2 = DN \times MC$$

(ii) In right triangle DBN,

$$\angle 5 + \angle 7 = 90^\circ \dots (4)$$

In right triangle DAN,

$$\angle 6 + \angle 8 = 90^\circ \dots (5)$$

D is the foot of the perpendicular drawn from B to AC.

$$\therefore \angle ADB = 90^\circ$$

$$\Rightarrow \angle 5 + \angle 6 = 90^\circ \dots (6)$$

From equation (4) and (6), we obtain

$$\angle 6 = \angle 7$$

From equation (5) and (6), we obtain

$$\angle 8 = \angle 5$$

In $\triangle DNA$ and $\triangle BND$,

$$\angle 6 = \angle 7 \text{ (Proved above)}$$

$$\angle 8 = \angle 5 \text{ (Proved above)}$$

$\therefore \triangle DNA \sim \triangle BND$ (AA similarity criterion)

$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

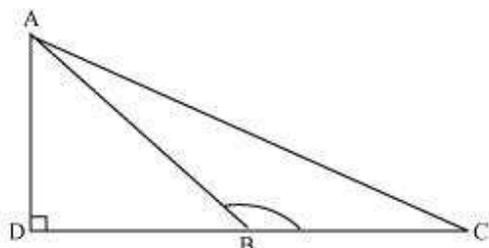
$$\Rightarrow DN^2 = AN \times NB$$

$$\Rightarrow DN^2 = AN \times DM \text{ (As } NB = DM)$$

Question 3:

In the given figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced.

Prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.



Answer:

Applying Pythagoras theorem in $\triangle ADB$, we obtain

$$AB^2 = AD^2 + DB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle ACD$, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = AD^2 + (DB + BC)^2$$

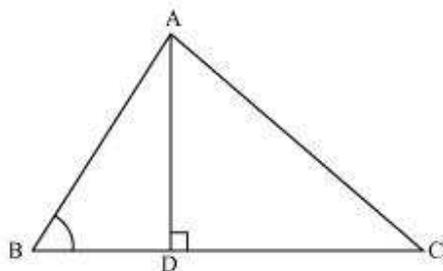
$$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$$

$$AC^2 = AB^2 + BC^2 + 2DB \times BC \text{ [Using equation (1)]}$$

Question 4:

In the given figure, ABC is a triangle in which $\angle ABC < 90^\circ$ and $AD \perp BC$. Prove that

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD.$$



Answer:

Applying Pythagoras theorem in $\triangle ADB$, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle ADC$, we obtain

$$AD^2 + DC^2 = AC^2$$



$$AB^2 - BD^2 + DC^2 = AC^2 \text{ [Using equation (1)]}$$

$$AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$$

$$= AB^2 + BC^2 - 2BC \times BD$$

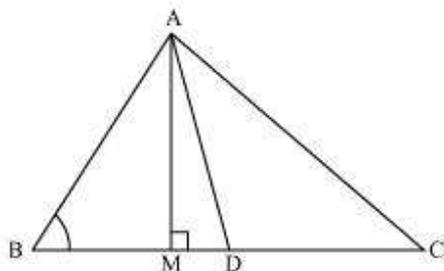
Question 5:

In the given figure, AD is a median of a triangle ABC and $AM \perp BC$. Prove that:

$$(i) \quad AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(ii) \quad AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(iii) \quad AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$



Answer:

(i) Applying Pythagoras theorem in $\triangle AMD$, we obtain

$$AM^2 + MD^2 = AD^2 \dots (1)$$

Applying Pythagoras theorem in $\triangle AMC$, we obtain

$$AM^2 + MC^2 = AC^2$$

$$AM^2 + (MD + DC)^2 = AC^2$$

$$(AM^2 + MD^2) + DC^2 + 2MD \cdot DC = AC^2$$

$$AD^2 + DC^2 + 2MD \cdot DC = AC^2 \text{ [Using equation (1)]}$$

Using the result, $DC = \frac{BC}{2}$, we obtain



$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD \cdot \left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in ΔABM , we obtain

$$AB^2 = AM^2 + MB^2$$

$$= (AD^2 - DM^2) + MB^2$$

$$= (AD^2 - DM^2) + (BD - MD)^2$$

$$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$$

$$= AD^2 + BD^2 - 2BD \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii) Applying Pythagoras theorem in ΔABM , we obtain

$$AM^2 + MB^2 = AB^2 \dots (1)$$

Applying Pythagoras theorem in ΔAMC , we obtain

$$AM^2 + MC^2 = AC^2 \dots (2)$$

Adding equations (1) and (2), we obtain

$$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$$

$$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$$

$$2AM^2 + BD^2 + DM^2 - 2BD \cdot DM + MD^2 + DC^2 + 2MD \cdot DC = AB^2 + AC^2$$

$$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$$

$$2(AM^2 + MD^2) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

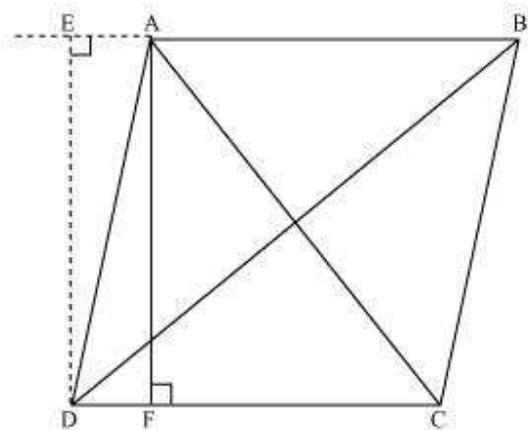
$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Question 6:



Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer:



Let ABCD be a parallelogram.

Let us draw perpendicular DE on extended side AB, and AF on side DC.

Applying Pythagoras theorem in $\triangle DEA$, we obtain

$$DE^2 + EA^2 = DA^2 \dots (i)$$

Applying Pythagoras theorem in $\triangle DEB$, we obtain

$$DE^2 + EB^2 = DB^2$$

$$DE^2 + (EA + AB)^2 = DB^2$$

$$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$$

$$DA^2 + AB^2 + 2EA \times AB = DB^2 \dots (ii)$$

Applying Pythagoras theorem in $\triangle ADF$, we obtain

$$AD^2 = AF^2 + FD^2$$

Applying Pythagoras theorem in $\triangle AFC$, we obtain

$$AC^2 = AF^2 + FC^2$$

$$= AF^2 + (DC - FD)^2$$

$$= AF^2 + DC^2 + FD^2 - 2DC \times FD$$

$$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$$

$$AC^2 = AD^2 + DC^2 - 2DC \times FD \dots (iii)$$



Since ABCD is a parallelogram,

$$AB = CD \dots (iv)$$

And, $BC = AD \dots (v)$

In $\triangle DEA$ and $\triangle ADF$,

$$\angle DEA = \angle AFD \text{ (Both } 90^\circ)$$

$$\angle EAD = \angle ADF \text{ (EA } \parallel \text{ DF)}$$

$$AD = AD \text{ (Common)}$$

$\therefore \triangle EAD \cong \triangle FDA$ (AAS congruence criterion)

$$\Rightarrow EA = DF \dots (vi)$$

Adding equations (i) and (iii), we obtain

$$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$$

$$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$$

$$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$$

[Using equations (iv) and (vi)]

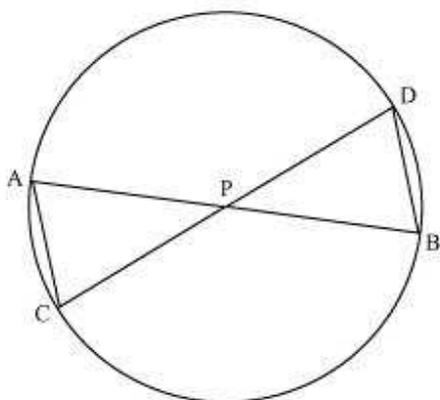
$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

Question 7:

In the given figure, two chords AB and CD intersect each other at the point P. prove that:

(i) $\triangle APC \sim \triangle DPB$

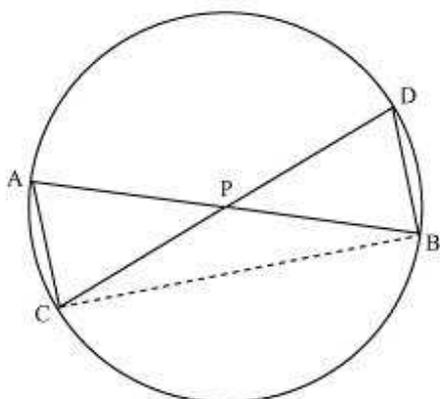
(ii) $AP \cdot BP = CP \cdot DP$





Answer:

Let us join CB.



(i) In $\triangle APC$ and $\triangle DPB$,

$\angle APC = \angle DPB$ (Vertically opposite angles)

$\angle CAP = \angle BDP$ (Angles in the same segment for chord CB)

$\triangle APC \sim \triangle DPB$ (By AA similarity criterion)

(ii) We have already proved that

$\triangle APC \sim \triangle DPB$

We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$$

$$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$$

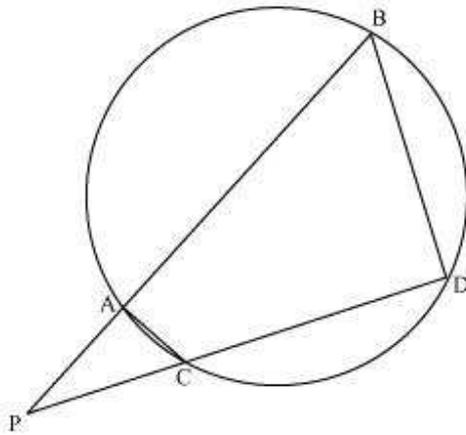
$$\therefore AP \cdot PB = PC \cdot DP$$

Question 8:

In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\triangle PAC \sim \triangle PDB$

(ii) $PA \cdot PB = PC \cdot PD$



Answer:

(i) In ΔPAC and ΔPDB ,

$\angle P = \angle P$ (Common)

$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is $\angle PCA = \angle PBD$ equal to the opposite interior angle)

$\therefore \Delta PAC \sim \Delta PDB$

(ii) We know that the corresponding sides of similar triangles are proportional.

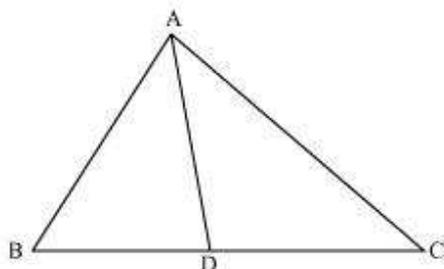
$$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$$

$$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$$

$$\therefore PA \cdot PB = PC \cdot PD$$

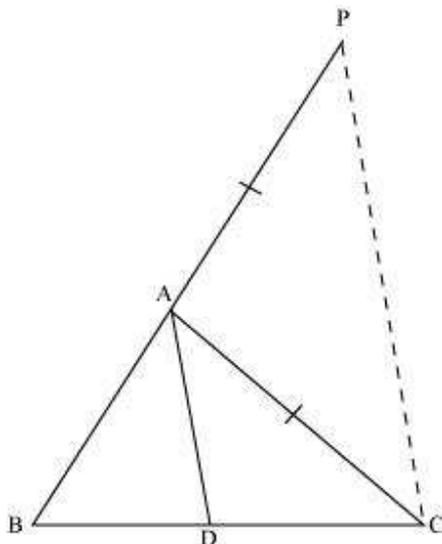
Question 9:

In the given figure, D is a point on side BC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of $\angle BAC$.



Answer:

Let us extend BA to P such that $AP = AC$. Join PC.



It is given that,

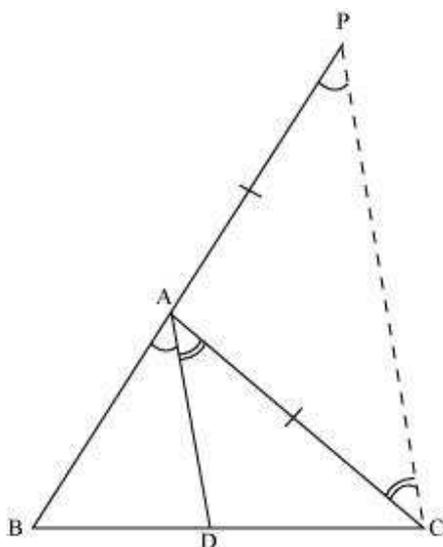
$$\frac{BD}{CD} = \frac{AB}{AC}$$
$$\Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

By using the converse of basic proportionality theorem, we obtain

$AD \parallel PC$

$\Rightarrow \angle BAD = \angle APC$ (Corresponding angles) ... (1)

And, $\angle DAC = \angle ACP$ (Alternate interior angles) ... (2)



By construction, we have

$$AP = AC$$

$$\Rightarrow \angle APC = \angle ACP \dots (3)$$

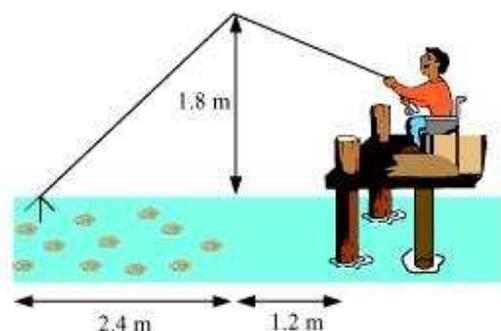
On comparing equations (1), (2), and (3), we obtain

$$\angle BAD = \angle APC$$

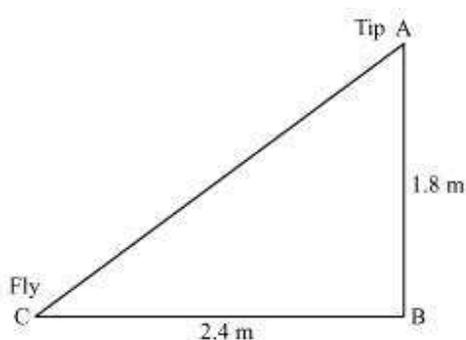
\Rightarrow AD is the bisector of the angle BAC

Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Answer:



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.

Then, AC is the length of the string.

AC can be found by applying Pythagoras theorem in ΔABC .

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = (1.8 \text{ m})^2 + (2.4 \text{ m})^2$$

$$AB^2 = (3.24 + 5.76) \text{ m}^2$$

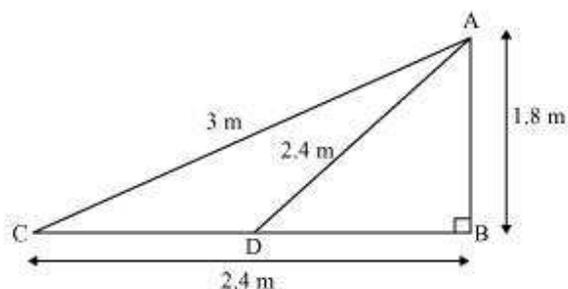
$$AB^2 = 9.00 \text{ m}^2$$

$$\Rightarrow AB = \sqrt{9} \text{ m} = 3 \text{ m}$$

Thus, the length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = $12 \times 5 = 60 \text{ cm} = 0.6 \text{ m}$



Let the fly be at point D after 12 seconds.

Length of string out after 12 seconds is AD.

$AD = AC - \text{String pulled by Nazima in 12 seconds}$

$$= (3.00 - 0.6) \text{ m}$$

$$= 2.4 \text{ m}$$

In $\triangle ADB$,

$$AB^2 + BD^2 = AD^2$$

$$(1.8 \text{ m})^2 + BD^2 = (2.4 \text{ m})^2$$

$$BD^2 = (5.76 - 3.24) \text{ m}^2 = 2.52 \text{ m}^2$$

$$BD = 1.587 \text{ m}$$

Horizontal distance of fly = $BD + 1.2 \text{ m}$

$$= (1.587 + 1.2) \text{ m}$$

$$= 2.787 \text{ m}$$

$$= 2.79 \text{ m}$$