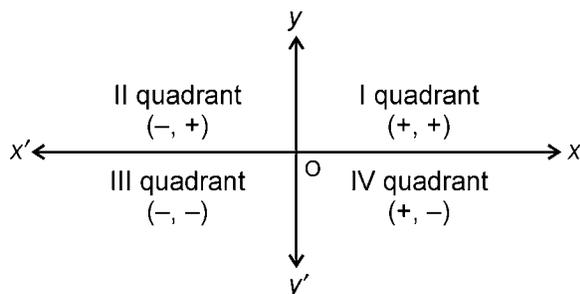


# Assignments in Mathematics Class X (Term II)

## 7. COORDINATE GEOMETRY

### IMPORTANT TERMS, DEFINITIONS AND RESULTS

- In the rectangular coordinate system, two number lines are drawn at right angles to each other. The point of intersection of these two number lines is called the **origin** whose coordinates are taken as (0, 0). The horizontal number line is known as the *x*-axis and the vertical one as the *y*-axis.
- In the ordered pair  $(p, q)$ ,  $p$  is called the *x*-coordinate or *abscissa* and  $q$  is known as *y*-coordinate or *ordinate* of the point.
- The coordinate plane is divided into four **quadrants**.



- The abscissa of a point is its perpendicular distance from *y*-axis.
- The ordinate of a point is its perpendicular distance from *x*-axis.
- The abscissa of every point situated on the right side of *y*-axis is **positive** and the abscissa of every point situated on the left side of *y*-axis is **negative**.
- The ordinate of every point situated above *x*-axis is **positive** and that of every point below *x*-axis is **negative**.
- The abscissa of every point on *y*-axis is zero.
- The ordinate of every point on *x*-axis is zero.
- The distance between any two points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{or } PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\Rightarrow PQ = \sqrt{(\text{Difference of abscissae})^2 + (\text{Difference of ordinates})^2}$$

- If  $O(0, 0)$  is the origin and  $P(x, y)$  is any point, then from the above formula, we have :

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

- In order to prove that a given figure is a :
  - (i) square, prove that four sides are equal and the diagonals are equal.
  - (ii) rhombus, prove that the four sides are equal.
  - (iii) rectangle, prove the opposite sides are equal

and the diagonals are also equal.

- (iv) parallelogram, prove that the opposite sides are equal.
  - (v) parallelogram but not a rectangle, prove that its opposite sides are equal but diagonals are not equal.
- Three points A, B and C are said to be collinear, if they lie on the same straight line.
  - For three points to be *collinear*, the sum of the distances between two pairs of points is equal to the third pair of points.
  - Three points will make :
    - (i) a scalene triangle, if no two sides of the triangle are equal.
    - (ii) an isosceles triangle, if any two sides are equal.
    - (iii) an equilateral triangle, if all the three sides are equal.
    - (iv) a right triangle, if sum of the squares of any two sides is equal to the square of the third side.

- The coordinates of the point  $P(x, y)$  which divides the line segment joining  $A(x_1, y_1)$  and  $B(x_2, y_2)$  internally in the ratio  $m : n$ , are given by :

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

- The coordinates of the mid-point M of a line segment AB with end points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  are :

$$\left[ \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right]$$

- The point of intersection of the medians of a triangle is called its *centroid*.
- The coordinates of the centroid of the triangle whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  are given by

$$\left[ \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right]$$

- The area of a  $\Delta ABC$  with vertices  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$  is given by :

area ( $\Delta ABC$ ) =

$$\left| \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} \right|$$

Since area of a triangle cannot be negative, we consider the absolute or numerical value of the area.

- Three given points  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$ , are collinear if

$$\Leftrightarrow \text{area of } \Delta ABC = 0$$

$$\Leftrightarrow \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Leftrightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

## SUMMATIVE ASSESSMENT

### MULTIPLE CHOICE QUESTIONS

[1 Mark]

#### A. Important Questions

1. Three points A, B, C are said to be collinear, if :
  - (a) they lie on the same straight line
  - (b) they do not lie on the same straight line
  - (c) they lie on three different straight lines
  - (d) none of these
2. The point  $(-3, 5)$  lies in :
  - (a) 1st quadrant
  - (b) 2nd quadrant
  - (c) 3rd quadrant
  - (d) 4th quadrant
3. The points  $A(0, -2)$ ,  $B(3, 1)$ ,  $C(0, 4)$  and  $D(-3, 1)$  are the vertices of a :
  - (a) parallelogram
  - (b) rectangle
  - (c) square
  - (d) rhombus
4. S is a point on  $x$ -axis at a distance of 4 units from  $y$ -axis to its right. The coordinates of S are :
  - (a)  $(4, 0)$
  - (b)  $(0, 4)$
  - (c)  $(4, 4)$
  - (d)  $(-4, 4)$
5. The distance between the points  $P(0, y)$  and  $Q(x, 0)$  is given by :
  - (a)  $x^2 + y^2$
  - (b)  $\sqrt{x^2 - y^2}$
  - (c)  $\sqrt{x^2 + y^2}$
  - (d)  $\sqrt{xy}$
6. The distance of the point  $P(2, 3)$  from the  $x$ -axis is :
  - (a) 2
  - (b) 3
  - (c) 1
  - (d) 5
7. The distance of the point  $P(-6, 8)$  from the origin is :
  - (a) 8
  - (b)  $2\sqrt{7}$
  - (c) 10
  - (d) 6
8. If the distance between the points  $(2, -2)$  and  $(-1, x)$  is 5, one of the values of  $x$  is :
  - (a)  $-2$
  - (b) 2
  - (c)  $-1$
  - (d) 1
9. The distance between the points  $(0, 5)$  and  $(5, 0)$  is :
  - (a) 5
  - (b)  $5\sqrt{2}$
  - (c)  $2\sqrt{5}$
  - (d) 10
10. The point on the  $x$ -axis which is equidistant from  $P(-2, 9)$  and  $Q(2, -5)$  is :
  - (a)  $(0, 7)$
  - (b)  $(-7, 0)$
  - (c)  $(7, 0)$
  - (d)  $(7, -7)$
11. The distance between the points  $P(2, -3)$  and  $Q(2, 2)$  is :
  - (a) 2 units
  - (b) 3 units
  - (c) 4 units
  - (d) 5 units
12. If the points  $P(2, 3)$ ,  $Q(5, k)$  and  $R(6, 7)$  are collinear, then the value of  $k$  is :
  - (a) 4
  - (b) 6
  - (c)  $-\frac{3}{2}$
  - (d)  $\frac{1}{4}$
13. The points  $M(0, 6)$ ,  $N(-5, 3)$  and  $P(3, 1)$  are the vertices of a triangle, which is :
  - (a) isosceles
  - (b) equilateral
  - (c) scalene
  - (d) right angled
14. A is a point on  $y$ -axis at a distance of 4 units from  $x$ -axis lying below  $x$ -axis. The coordinates of A are :
  - (a)  $(4, 0)$
  - (b)  $(0, 4)$
  - (c)  $(-4, 0)$
  - (d)  $(0, -4)$
15. The mid-point of the line segment joining the points  $A(-2, 8)$  and  $B(-6, -4)$  is :
  - (a)  $(-6, -4)$
  - (b)  $(2, 6)$
  - (c)  $(-4, 2)$
  - (d)  $(4, 2)$
16. The point which divides the line segment joining the points  $(7, -6)$  and  $(3, 4)$  in the ratio  $1 : 2$  internally lies in the :
  - (a) 1st quadrant
  - (b) 2nd quadrant
  - (c) 3rd quadrant
  - (d) 4th quadrant
17. If the point  $P(2, 1)$  lies on the line segment joining points  $A(4, 2)$  and  $B(8, 4)$ , then :
  - (a)  $AP = \frac{AE}{3}$
  - (b)  $AP = AB$
  - (c)  $PB = \frac{AE}{3}$
  - (d)  $AP = \frac{AE}{2}$
18. If the points  $(k, 2k)$ ,  $(3k, 3k)$  and  $(3, 1)$  are collinear, then  $k$  is :
  - (a)  $\frac{1}{3}$
  - (b)  $-\frac{1}{3}$
  - (c)  $\frac{2}{3}$
  - (d)  $-\frac{2}{3}$

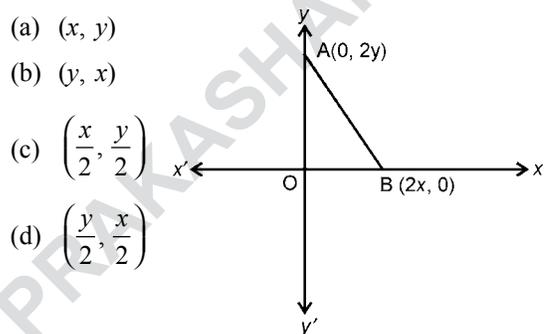
19. The points  $(0, 6)$ ,  $(-5, 3)$  and  $(3, 1)$  are the vertices of a triangle which is :
- (a) equilateral (b) isosceles  
(c) scalene (d) right angled
20. If  $P\left(\frac{a}{3}, 4\right)$  is the mid-point of the line segment joining the point  $Q(-6, 5)$  and  $R(-2, 3)$ , then the value of  $a$  is :
- (a)  $-4$  (b)  $-12$  (c)  $12$  (d)  $-6$
21. The perpendicular bisector of the line segment joining the points  $A(1, 5)$  and  $B(4, 6)$  cuts the  $y$ -axis at:
- (a)  $(0, 13)$  (b)  $(0, -13)$   
(c)  $(0, 12)$  (d)  $(13, 0)$
22. The ratio in which  $(4, 5)$  divides the join of  $(2, 3)$  and  $(7, 8)$  is :
- (a)  $4 : 3$  (b)  $5 : 2$   
(c)  $3 : 2$  (d)  $2 : 3$
23. The  $y$ -axis divides the join of  $P(-4, 2)$  and  $Q(8, 3)$  in the ratio :
- (a)  $3 : 1$  (b)  $1 : 3$   
(c)  $2 : 1$  (d)  $1 : 2$
24. Two vertices of  $\Delta PQR$  are  $P(-1, 4)$  and  $Q(5, 2)$  and its centroid is  $G(0, -3)$ . The coordinates of  $R$  are:
- (a)  $(4, 3)$  (b)  $(4, 15)$   
(c)  $(-4, -15)$  (d)  $(-15, -4)$
25. The  $x$ -axis divides the join of  $A(2, -3)$  and  $B(5, 6)$  in the ratio :
- (a)  $1 : 2$  (b)  $2 : 1$   
(c)  $3 : 5$  (d)  $2 : 3$
26. A point  $A$  divides the join of  $X(5, -2)$  and  $Y(9, 6)$  in the ratio  $3 : 1$ . The coordinates of  $A$  are :
- (a)  $(4, 7)$  (b)  $(8, 4)$   
(c)  $\left(\frac{11}{2}, 5\right)$  (d)  $(12, 8)$
27. If  $M(-1, 1)$  is the mid-point of the line segment joining  $P(-3, y)$  and  $Q(1, y + 4)$ , then the value of  $y$  is:
- (a)  $1$  (b)  $-1$  (c)  $2$  (d)  $0$
28. If  $(x, 2)$ ,  $(-3, -4)$  and  $(7, -5)$  are collinear, then  $x$  is equal to :
- (a)  $60$  (b)  $63$  (c)  $-63$  (d)  $-60$
29. If the area of the triangle formed by the points  $(a, 2a)$ ,  $(-2, 6)$  and  $(3, 1)$  is 5 square units, then  $a$  is equal to :
- (a)  $2$  (b)  $\frac{3}{5}$  (c)  $3$  (d)  $5$
30. If the distance between the points  $(4, p)$  and  $(1, 0)$  is 5, then the value of  $p$  is :
- (a) 4 only (b)  $\pm 4$   
(c)  $-4$  only (d) 0
31. A line intersects  $x$  and  $y$ -axes at  $P$  and  $Q$  respectively. If  $(2, -5)$  is the mid-point of  $PQ$ , then the coordinates of  $P$  and  $Q$  are respectively:
- (a)  $(4, 0)$  and  $(0, -10)$  (b)  $(2, 0)$  and  $(0, -5)$   
(c)  $(-4, 0)$  and  $(0, 10)$  (d)  $(-10, 0)$  and  $(0, 4)$
32. The distance between the points  $(\cos\theta, \sin\theta)$  and  $(\sin\theta, \cos\theta)$  is :
- (a)  $\sqrt{3}$  (b)  $\sqrt{2}$  (c) 2 (d) 1
33. If the points  $(1, 2)$ ,  $(-5, 6)$  and  $(a, -2)$  are collinear, then  $a$  is equal to :
- (a)  $-3$  (b) 7 (c) 2 (d)  $-2$
34. The points  $A(9, 0)$ ,  $B(9, 6)$ ,  $C(-9, 6)$  and  $D(-9, 0)$  are the vertices of a :
- (a) square (b) rectangle  
(c) rhombus (d) trapezium
35.  $AOBC$  is a rectangle whose three vertices are  $A(0, 3)$ ,  $O(0, 0)$  and  $B(5, 0)$ . The length of its diagonal is :
- (a) 5 (b) 3 (c)  $\sqrt{34}$  (d) 4
36. The points  $(-4, 0)$ ,  $(4, 0)$  and  $(0, 3)$  are vertices of a:
- (a) right triangle (b) isosceles triangle  
(c) equilateral triangle (d) scalene triangle
37. The coordinates of the vertices of an equilateral triangle are  $A(3, y)$ ,  $B(3, \sqrt{3})$  and  $C(0, 0)$ . The value of  $y$  is :
- (a) 4 (b) 5  
(c)  $-1$  (d) none of these
38. The distance between the points  $(a \cos \theta + b \sin \theta, 0)$  and  $(0, a \sin \theta - b \cos \theta)$  is :
- (a)  $a^2 + b^2$  (b)  $a + b$   
(c)  $a^2 - b^2$  (d)  $\sqrt{a^2 + b^2}$
39.  $(-1, 2)$ ,  $(2, -1)$  and  $(3, 1)$  are three vertices of a parallelogram. The coordinates of the fourth vertex are:
- (a)  $(4, 0)$  (b)  $(-2, 0)$   
(c)  $(-2, 6)$  (d)  $(6, 2)$
40. If  $A(5, 3)$ ,  $B(11, -5)$  and  $C(12, a)$  are the vertices of a right angled triangle, right angled at  $C$ , then the value of  $a$  is :
- (a)  $-2, 4$  (b)  $-2, -4$   
(c)  $2, -4$  (d)  $2, 4$

41. The perimeter of the triangle with vertices (0, 4), (0, 0) and (3, 0) is :  
 (a) 5 (b) 12  
 (c) 11 (d)  $7 + \sqrt{5}$
42. The point which lies on the perpendicular bisector of the line segment joining the points A(-2, -5) and B(2, 5) is :  
 (a) (0, 0) (b) (0, 2)  
 (c) (2, 0) (d) (-2, 0)
43. The area of the triangle with vertices (a, b + c), (b, c + a) and (c, a + b) is :  
 (a)  $(a + b + c)^2$  (b) 0  
 (c)  $(a + b + c)$  (d)  $abc$
44. If the centroid of the triangle formed by the points (a, b), (b, c) and (c, a) is at the origin, then  $a^3 + b^3 + c^3$  is equal to :  
 (a)  $abc$  (b) 0  
 (c)  $a + b + c$  (d)  $3abc$
45. The line segment joining points (-3, -4) and (1, -2) is divided by y-axis in the ratio :  
 (a) 1 : 3 (b) 2 : 3  
 (c) 3 : 1 (d) 2 : 3
46. If points (x, 0), (0, y) are (1, 1) and collinear, then  $\frac{1}{x} + \frac{1}{y}$  is equal to :  
 (a) 1 (b) 2 (c) 0 (d) -1
47. If the ratio in which P divides the line segment joining  $(x_1, y_1)$  and  $(x_2, y_2)$  be  $k : 1$ , then the coordinates of the point P are :  
 (a)  $\left(\frac{kx_1 + x_2}{k+1}, \frac{ky_1 + y_2}{k+1}\right)$   
 (b)  $\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$   
 (c)  $\left(\frac{x_1 + x_2}{k+1}, \frac{y_1 + y_2}{k+1}\right)$   
 (d) none of these
48. Area of a triangle is taken :  
 (a) always positive  
 (b) always negative  
 (c) sometimes positive sometimes negative  
 (d) none of these
49. The area of the triangle formed by (x, y + z), (y, z + x) and (z, x + y) is :  
 (a)  $x + y + z$  (b)  $xyz$

(c)  $(x + y + z)^2$  (d) 0

50. The fourth vertex D of a parallelogram ABCD whose three vertices are A(-2, 3), B(6, 7) and C(8, 3) is :  
 (a) (0, 1) (b) (0, -1)  
 (c) (-1, 0) (d) (1, 0)
51. If the points A(1, 2), B(0, 0) and C(a, b) are collinear, then :  
 (a)  $a = b$  (b)  $a = 2b$   
 (c)  $2a = b$  (d)  $a = -b$

52. In the figure, OAB is a triangle. The coordinates of the point which is equidistant from the three vertices are :



(a) (x, y)

(b) (y, x)

(c)  $\left(\frac{x}{2}, \frac{y}{2}\right)$

(d)  $\left(\frac{y}{2}, \frac{x}{2}\right)$

53. A circle drawn with origin as the centre passes through  $\left(\frac{13}{2}, 0\right)$ . The point which does not lie in the interior of the circle is :

(a)  $\left(-\frac{3}{4}, 1\right)$

(b)  $\left(2, \frac{7}{3}\right)$

(c)  $\left(5, \frac{1}{2}\right)$

(d)  $\left(-6, \frac{5}{2}\right)$

54. The coordinates of the centroid of the triangle with vertices (a, 0), (0, b) and (a, b) are :

(a)  $\left(\frac{a}{2}, \frac{b}{2}\right)$

(b)  $\left(\frac{a}{3}, \frac{b}{3}\right)$

(c)  $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

(d) none of these

55. Area of the triangle with vertices (x, 0), (0, y) and (x, y) is :

(a)  $x + y$

(b)  $x - y$

(c)  $\frac{x}{y}$

(d)  $\frac{xy}{z}$

56. If the area of a quadrilateral ABCD is zero, then the four points A, B, C and D are :

(a) collinear

(b) not collinear

(c) nothing can be said

(d) none of these

## B. Questions From CBSE Examination Papers

- If the points  $(0, 0)$ ,  $(1, 2)$  and  $(x, y)$  are collinear then : [2011 (T-II)]  
(a)  $x = y$  (b)  $2x = y$   
(c)  $x = 2y$  (d)  $2x = -y$
- The perpendicular distance of A  $(5, 12)$  from the  $y$ -axis is : [2011 (T-II)]  
(a) 13 units (b) 5 units  
(c) 12 units (d) 17 units
- The perimeter of a triangle with vertices  $(0, 4)$ ,  $(0, 0)$  and  $(3, 0)$  is : [2011 (T-II)]  
(a) 8 (b) 10 (c) 12 (d) 15

## SHORT ANSWER TYPE QUESTIONS

[2 Marks]

### A. Important Questions

- Find the distance between the points  $(a \cos 35^\circ, 0)$  and  $(0, a \cos 55^\circ)$
- Find the value of  $x$  which is an integer such that the distance between the points  $P(x, 2)$  and  $Q(3, -6)$  is 10 units.
- Is the point  $(4, 4)$  equidistant from the points  $P(-1, 4)$  and  $Q(1, 0)$ ?
- A is a point on the  $x$ -axis and B is a point on the  $y$ -axis. If the abscissa of A be  $a$  and the ordinate of B be  $-a$ , then find the length of segment AB.
- What is the distance between A on the  $x$ -axis whose abscissa is 11 and  $B(7, 3)$ ?
- Find the coordinates of the other end of a diameter of a circle whose one end is  $A(2, 1)$  and centre is  $P\left(\frac{3}{2}, \frac{-5}{2}\right)$ .
- Find the coordinates of the centroid of the triangle whose vertices are  $(0, 6)$ ,  $(8, 12)$  and  $(8, 0)$ .
- If the point  $(a, b)$  is equidistant from the points  $(7, 1)$  and  $(3, 5)$ , find the relation between  $a$  and  $b$ .
- Two vertices of a triangle are  $A(-7, 4)$  and  $B(3, -5)$ . If its centroid is  $(2, -1)$ , then find the coordinates of the third vertex C.
- The mid-point of the line segment joining  $(3, 6)$  and  $(x, 2)$  is  $(2, y)$ . Find the values of  $x$  and  $y$ .
- What point on the  $x$ -axis is equidistant from  $(7, 6)$  and  $(-3, 4)$ ?
- Find the ratio in which the line segment joining the points  $(6, 4)$  and  $(1, -7)$  is divided by the  $x$ -axis.
- Find the distance between the points  $A(x + y, x - y)$  and  $B(x - y, -x - y)$ .
- Are the points  $A(a, b + c)$ ,  $B(b, c + a)$  and  $C(c, a + b)$  collinear?
- Find the value of  $x$  if the points  $(x, 8)$ ,  $(-4, 2)$  and  $(5, -1)$  are collinear.
- Is  $\triangle ABC$  with vertices  $A(-2, 0)$ ,  $B(2, 0)$ ,  $C(0, 2)$  similar to  $\triangle DEF$  with vertices  $D(-4, 0)$ ,  $E(4, 0)$  and  $F(0, 4)$ ? Justify your answer.
- Find the points on the  $y$ -axis, each of which is at a distance of 13 units from the point  $(-5, 7)$ .
- Are the points  $A(4, 5)$ ,  $B(7, 6)$  and  $C(6, 3)$  collinear?
- If the points  $P(a, -11)$ ,  $Q(5, b)$ ,  $R(2, 15)$  and  $S(1, 1)$  are the vertices of a parallelogram PQRS, find the values of  $a$  and  $b$ .
- Find all possible values of  $a$  for which the distance between the points  $A(a, -1)$  and  $B(5, 3)$  is 5 units.
- Show that the points  $A(3, 1)$ ,  $B(12, -2)$  and  $C(0, 2)$  cannot be the vertices of a triangle.
- If the points  $A(-6, 10)$ ,  $B(-4, 6)$  and  $C(3, -8)$  are collinear, then show that  $AB = \frac{2}{9} AC$ .
- Find the coordinates of the point R which divides the line segment joining the points  $P(-2, 3)$  and  $Q(4, 7)$  internally in the ratio 4 : 7.
- If the point  $C(-1, 2)$  divides internally the line segment joining  $A(2, 5)$  and B in the ratio 3 : 4, find the coordinates of B.
- Check whether the point  $P(-2, 4)$  lies on a circle of radius 6 units and centre  $C(3, 5)$ .
- Does the point  $A(-4, 2)$  lie on the line segment joining the points  $X(-4, 6)$  and  $Y(-4, -6)$ ? Justify your answer.
- In what ratio does the point  $P(2, -5)$  divide the line segment joining  $A(-3, 5)$  and  $B(4, -9)$ ?
- Show that the points  $A(-1, -2)$ ,  $B(4, 3)$ ,  $C(2, 5)$  and  $D(-3, 0)$  are the vertices of the rectangle ABCD.
- Show that  $A(-2, 3)$ ,  $B(8, 3)$  and  $C(6, 7)$  are the vertices of a right angled triangle.
- Find the value of  $m$  if the point  $(0, 2)$  is equidistant from  $(3, m)$  and  $(m, 5)$ .

## B. Questions From CBSE Examination Papers

- If the point A (4, 3) and B(x, 5) are on the circle with centre O(2, 3), find the value of x. [2011 (T-II)]
- Three consecutive vertices of a parallelogram ABCD are A(1, 2), B(1, 0) and C(4, 0). Find the fourth vertex D. [2011 (T-II)]
- Which point on x-axis is equidistant from (7, 6) and (-3, 4)? [2011 (T-II)]
- Find the ratio in which the line  $2x + y = 4$  divides the join of A(2, -2) and B(3, 7). Also, find the coordinates of the point of their intersection. [2011 (T-II)]
- Find the value of  $k$  for which the distance between (9, 2) and (3,  $k$ ) is 10 units. [2011 (T-II)]
- Find the ratio in which the point  $\left(\frac{-2}{7}, \frac{-20}{7}\right)$  divides the join of (-2, -2) and (2, -4). [2011 (T-II)]
- Find the coordinates of a point on x-axis which divides the line segment joining the points (-2, -3) and (1, 6) in the ratio 1 : 2. [2011 (T-II)]
- The line segment joining the points P(3, 3) and Q(6, -6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by  $2x + y + k = 0$ , find the value of  $k$ .
- Find the type of triangle formed by points A(-5, 6), B(-4, -2), C(7, 5). [2011 (T-II)]
- In what ratio does the point (-4, 6) divide the line segment joining the points A(-6, 10) and B(3, -8)? [2011 (T-II)]
- Find a relation between  $x$  and  $y$  such that the point P(x, y) is equidistant from the points A(7, 1) and B(3, 5). [2011 (T-II)]
- If the points A(6, 1), B(8, 2), C(9, 4) and D(P, 3) are the vertices of a parallelogram taken in order, find the value of P. [2011 (T-II)]
- Find the co-ordinates of the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4). [2011 (T-II)]
- Check whether the points (2, 2), (4, 0) and (-6, 10) are collinear. [2011 (T-II)]
- Find the ratio in which the y-axis divides the join of (5, -6) and (-1, -4). [2011 (T-II)]
- Find the co-ordinates of a point A, where AB is the diameter of a circle whose centre is O(2, -3) and B is (1, 4). [2011 (T-II)]
- Using section formula, show that the points A(-3, 1), B(1, 3) and C(-1, 1) are collinear.
- Find the value of  $p$ , for which the points (1, 3), (3,  $p$ ) and (5, -1) are collinear. [2011 (T-II)]
- Find points on the x-axis, which are at a distance of 5 units from the point A (5, -3). [2011 (T-II)]
- Show that the points ( $a, b + c$ ), ( $b, c + a$ ) and ( $c, a + b$ ) are collinear. [2011 (T-II)]
- Prove that the points (0, 0), (5, 5) and (-5, 5) are the vertices of a right angled isosceles triangle.
- Find the value of  $x$ , if the distance between the points ( $x, -1$ ) and (3, -2) is  $x + 5$ . [2011 (T-II)]
- If the point C (-1, 2) divides internally the line segment joining A (2, 5) and B(x, y) in the ratio 3 : 4, then find the coordinates of B. [2011 (T-II)]
- Show that the point P (-4, 2) lies on the line segment joining the points A (-4, 6) and B (-4, -6). [2011 (T-II)]
- Show that the point (1, -1) is the centre of the circle circumscribing the triangle whose vertices are (4, 3) and (-2, 3) and (6, -1). [2011 (T-II)]
- If A(1, 2), B (4, y), C(x, 6) and D(3, 5) are the vertices of a parallelogram ABCD taken in order, find the values of  $x$  and  $y$ . [2011 (T-II)]
- If the point P(x, y) is equidistant from the points A(5, 1) and B (-1, 5) then prove that  $3x = 2y$ . [2011 (T-II)]
- Determine the ratio in which the point P(x, -2) divides the join of A(-4, 3) and B(2, -4). Also find the value of  $x$ . [2011 (T-II)]
- If points A(-2, -1), B ( $a, 0$ ), C (4,  $b$ ) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of  $a$  and  $b$ . [2011 (T-II)]
- Find the value of  $x$  such that PQ = QR, where the coordinates of P, Q and R are (6, -1), (1, 3) and ( $x, 8$ ) respectively. [2011 (T-II)]
- Find the value of  $k$  for which the point (8, 1), ( $k, -4$ ) and (2, -5) are collinear. [2011 (T-II)]
- Show that P(1, -1) is the centre of the circle circumscribing the triangle whose angular points are A (4, 3), B(-2, 3) and C (6, -1). [2011 (T-II)]
- One end of a diameter of a circle is at (2, 3) and the centre is (-2, 5). What are the coordinates of the other end of the diameter? [2011 (T-II)]
- A point P is at a distance of  $\sqrt{10}$  from the point (2, 3). Find the coordinates of the point P if its y coordinate is twice its x coordinate. [2011 (T-II)]
- Find the coordinates of the point B, if the point

P(-4, 1) divides the line segment joining the points A(2, -2) and B in the ratio 3 : 5.

[2011 (T-II)]

36. Find the third vertex of the triangle ABC if two of its vertices are at A(-3, 1) and B (0, 2) and the mid-point of BC is at  $D\left(\frac{3}{2}, -\frac{1}{2}\right)$ .

[2011 (T-II)]

37. Find the value of  $s$  if the point P(0, 2) is equidistant from Q (3,  $s$ ) and R( $s$ , 5).
38. Find the perimeter of the triangle formed by the points (0, 0), (1, 0), (0, 1). [2011 (T-II)]

39. Find the points on the  $x$ -axis which are at a distance of  $2\sqrt{5}$  from the point (7, -4). How many such points are there? [2011 (T-II)]

40. If A and B are the points (-2, -2) and (2, -4) respectively, find the coordinates of P on the line segment AB such that  $AP = \frac{3}{7} AB$ .

[2011 (T-II)]

41. Find a point on  $x$ -axis which is equidistant from (-3, -4) and (4, -3). [2011 (T-II)]

42. Find the value of  $a$  so that the point (3,  $a$ ) lies on the line represented by  $2x - 3y = 5$ . [2009]

## SHORT ANSWER TYPE QUESTIONS

[3 Marks]

### A. Important Questions

1. If P( $x$ ,  $y$ ) is the mid-point of the line segment joining the points A(3, 4) and B( $k$ , 6) and  $x + y = 10$ , then find the value of  $k$ .

2. Find the coordinates of the centre of a circle passing through the points A(2, 1), B(5, -8) and C(2, -9). Also, find the radius of the circle.

3. Find the points on the  $x$ -axis which are at a distance of  $2\sqrt{5}$  units from the point (7, -4). How many such points are there ?

4. Find the area of the triangle ABC with A(1, -4) and the mid-points of the sides through A being (2, -1) and (0, -1).

5. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

6. If three points ( $x_1, y_1$ ), ( $x_2, y_2$ ) and ( $x_3, y_3$ ) lie on the same line, prove that [HOTS]

$$\frac{y_2 - y_3}{x_2 x_3} + \frac{y_3 - y_1}{x_3 x_1} + \frac{y_1 - y_2}{x_1 x_2} = 0$$

7. The centre of a circle is (2a,  $a - 7$ ). Find the value of  $a$ , if the circle passes through the point (11, -9) and has diameter  $10\sqrt{2}$  units.

8. What type of triangle is formed by the points A( $\sqrt{2}, \sqrt{2}$ ) B( $\sqrt{2}, -\sqrt{2}$ ) and C( $-\sqrt{6}, \sqrt{6}$ ).

9. Find the ratio in which the point M  $\left(\frac{3}{4}, \frac{5}{12}\right)$  divides the line segment joining the points A  $\left(\frac{1}{2}, \frac{3}{2}\right)$  and B(2, -5).

10. If the mid-points of the sides of a triangle are (2, 3),  $\left(\frac{3}{2}, 4\right)$  and  $\left(\frac{11}{2}, 5\right)$ , find the centroid of the triangle.

11. Using coordinate geometry, prove that the diagonals of a rectangle bisect each other and are equal.

12. Find the coordinates of the points C on the line segment joining the points A(-1, 3) and B(2, 5) such that  $AC = \frac{3}{5} AB$ .

13. Show that (4, -1), (6, 0), (7, 2) and (5, 1) are the vertices of a rhombus. Is it a square?

14. The coordinates of the three vertices A, B and C of a parallelogram ABCD are ( $x_1, y_1$ ), ( $x_2, y_2$ ) and ( $x_3, y_3$ ) respectively. Find the coordinates of the fourth vertex D in terms of  $x_1, x_2, x_3, y_1, y_2, y_3$ .

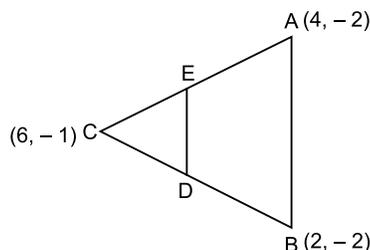
[HOTS]

15. If D  $\left(-\frac{1}{2}, \frac{5}{2}\right)$ , E(7, 3) and F  $\left(\frac{7}{2}, \frac{7}{2}\right)$  are the mid-points of sides of  $\Delta ABC$ , find the area of  $\Delta ABC$ .

16. If the line segment joining the points A(3a + 1, -3) and B(8a, 5) is divided by the point P(9a - 2, -b) in the ratio 3 : 1, find the values of  $a$  and  $b$ .

## B. Questions From CBSE Examination Papers

1. Find the value(s) of  $x$  for which distance between the points  $P(2, -3)$  and  $Q(x, 5)$  is 10 units. [2011 (T-II)]
2. Find a relation between  $x$  and  $y$  such that the point  $P(x, y)$  is equidistant from the points  $A(2, 5)$  and  $B(-3, 7)$ . [2011 (T-II)]
3. Find the area of a triangle  $ABC$  whose vertices are  $A(1, -1)$ ,  $B(-4, 6)$  and  $C(-3, -5)$ .
4. If the points  $(1, 4)$ ,  $(r, -2)$  and  $(-3, 16)$  are collinear, find the value of ' $r$ '. [2011 (T-II)]
5. If  $A(3, 0)$ ,  $B(4, 5)$ ,  $C(-1, 4)$  and  $D(-2, -1)$  be four points in a plane, show that  $ABCD$  is a rhombus but not a square. [2011 (T-II)]
6. The mid-point of the line segment joining points  $A(x, y + 1)$  and  $B(x + 1, y + 2)$  is  $C$ . Find the value of  $x$  and  $y$  if the coordinates of  $C$  are  $(3/2, 5/2)$ . [2011 (T-II)]
7. Find the area of rhombus  $ABCD$  if its vertices are  $A(3, 0)$ ,  $B(4, 5)$ ,  $C(-1, 4)$ ,  $D(-2, -1)$ . [2011 (T-II)]
8. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are  $(0, -1)$ ,  $(2, 1)$  and  $(0, 3)$ . [2011 (T-II)]
9. The area of a triangle whose vertices are  $(-2, -2)$ ,  $(-1, -3)$  and  $(x, 0)$  is 3 square units. Find the value of  $x$ . [2011 (T-II)]
10. If  $P(2, 1)$ ,  $Q(4, 2)$ ,  $R(5, 4)$  and  $S(3, 3)$  are vertices of a quadrilateral  $PQRS$ , find the area of the quadrilateral  $PQRS$ . [2011 (T-II)]
11. If  $(1, 2)$ ,  $(4, y)$ ,  $(x, 6)$  and  $(3, 5)$  are the vertices of a parallelogram taken in order, find  $x$  and  $y$ .
12. Show that the points  $P(0, -2)$ ,  $Q(3, 1)$ ,  $R(0, 4)$  and  $S(-3, 1)$  are the vertices of a square.  $PQRS$ . [2011 (T-II)]
13. The points  $A(2, 9)$ ,  $B(a, 5)$  and  $C(5, 5)$  are the vertices of a triangle  $ABC$ , right angled at  $B$ . Find the value of  $a$  and hence the area of triangle  $ABC$ . [2011 (T-II)]
14. Find the value of  $k$  for which the points  $A(-1, 3)$ ,  $B(2, k)$  and  $C(5, -1)$  are collinear. [2011 (T-II)]
15. Find the lengths of the medians  $AD$  and  $BE$  of the triangle  $ABC$  whose vertices are  $A(1, -1)$ ,  $B(0, 4)$  and  $C(-5, 3)$ . [2011 (T-II)]
16. The mid-points of the sides  $AB$ ,  $BC$  and  $CA$  of a triangle  $ABC$  are  $D(2, 1)$ ,  $E(1, 0)$  and  $F(-1, 3)$  respectively. Find the coordinates of the vertices of the triangle  $ABC$ . [2011 (T-II)]
17.  $ABCD$  is a rectangle formed by joining the points  $A(-1, -1)$ ,  $B(-1, 4)$ ,  $C(5, 4)$  and  $D(5, -1)$ .  $P$ ,  $Q$ ,  $R$  and  $S$  are the mid-points of  $AB$ ,  $BC$ ,  $CD$  and  $DA$  respectively. Is the quadrilateral  $PQRS$  a square, a rectangle or a rhombus? Justify your answer. [2011 (T-II)]
18. The line segment joining the points  $A(2, 1)$  and  $B(5, -8)$  is trisected at the points  $P$  and  $Q$ , where  $P$  is nearer to  $A$ . If point  $P$  lies on the line  $2x - y + k = 0$ , find the value of  $k$ . [2011 (T-II)]
19. If  $P(x, y)$  is any point on the line segment joining the points  $A(a, 0)$  and  $B(0, b)$ , then show that 
$$\frac{x}{a} + \frac{y}{b} = 1$$
 [2011 (T-II)]
20. If  $C$  is a point lying on the line segment  $AB$  joining  $A(1, 1)$ ,  $B(2, 3)$  such that  $3AC = BC$ , then find co-ordinates of  $C$ . [2011 (T-II)]
21. In the figure, in  $\triangle ABC$ ,  $D$  and  $E$  are the mid-points of the sides  $BC$  and  $AC$  respectively. Find the length of  $DE$ . Prove that  $DE = \frac{1}{2} AB$ . [2011 (T-II)]



27. Point P divides the line segment joining the points A(2, 1) and B(5, -8) such that  $\frac{AP}{PB} = \frac{1}{3}$ . If P lies on the line  $2x - y + k = 0$ , find the value of  $k$ . [2011 (T-II)]
28. If the vertices of a triangle are (2, 4), (5, k), (3, 10) and its area is 15 square units, find value of  $k$ . [2011 (T-II)]
29. Find the coordinates of centre of the circle passing through the point (0, 0), (-2, 1) and (-3, 2). Also find its radius. [2011 (T-II)]
30. If mid-points of sides  $\Delta PQR$  are (1, 2), (0, 1), (1, 0), then find the coordinates of the three vertices of triangle PQR. [2011 (T-II)]
31. Prove that the points (0, 0), (5, 5), (-5, 5) are the vertices of a right angled triangle. [2011 (T-II)]
32. Find the co-ordinates of the points which divide the line segment joining A(-2, 2) and B(2, 8) into four equal parts. [2011 (T-II)]
33. If A(4, -6), B(3, -2) and C(5, 2) are the vertices of a  $\Delta ABC$ , then verify the fact that a median of  $\Delta ABC$  divides it into two triangles of equal areas. [2011 (T-II)]
34. If the points (10, 5), (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices. [2006]
35. In what ratio does the line  $x - y - 2 = 0$  divide the line segment joining (3, -1) and (8, 9)? [2007]
36. Three consecutive vertices of a parallelogram are (-2, 1), (1, 0) and (4, 3). Find the coordinate of the fourth vertex. [2007]
37. The point R divides the line segment AB where A(-4, 0), B(0, 6) are such that  $AR = \frac{3}{4} AB$ . Find the coordinates of R. [2008]
38. The coordinates of A and B are (1, 2) and (2, 3) respectively. If P lies on AB, find the coordinates of P such that  $\frac{AP}{PB} = \frac{4}{3}$ . [2008]
39. If A(4, -8), B(3, 6) and C(5, -4) are the vertices of  $\Delta ABC$ , D is the mid-point of BC and P is a point on AD joined such that  $\frac{AP}{PD} = 2$ . Find the coordinates of P. [2008]
40. The line segment joining the points A(2, 1) and B(5, -8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by  $2x - y + k = 0$ , find the value of  $k$ . [2009]
41. If P(x, y) is any point on the line joining the points A(a, 0) and B(0, b), then show that  $\frac{x}{a} + \frac{y}{b} = 1$ . [2009]
42. If the points (p, q), (m, n) and (p - m, q - n) are collinear, show that  $pn = qm$ . [2010]
43. Point P divides the line segment joining the points A(-1, 3) and B(9, 8) such that  $\frac{AP}{PB} = \frac{k}{1}$ . If P lies on the line  $x - y + 2 = 0$ , find the value of  $k$ . [2010]

## LONG ANSWER TYPE QUESTIONS

[4 Marks]

### A. Important Questions

1. The points A( $x_1, y_1$ ), B( $x_2, y_2$ ) and C( $x_3, y_3$ ) are the vertices of  $\Delta ABC$ .
- The median from A meets BC at D. What are the coordinates of the point D?
  - Find the coordinates of the point P on AD such that  $AP : PD = 2 : 1$ .
  - Find the coordinates of points Q and R on medians BE and CF respectively such that  $BQ : QE = 2 : 1$  and  $CR : RF = 2 : 1$ .
  - What are the coordinates of the centroid of the triangle ABC? [HOTS]
2. If P and Q are two points whose coordinates are ( $at^2, 2at$ ) and  $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$  respectively and S is the point (a, 0), show that  $\frac{1}{SP} + \frac{1}{SQ}$  is independent of  $t$ . [HOTS]
3. If (-4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex given that the origin lies in the interior of the triangle.
4. If the points (x, y), ( $x_1, y_1$ ) and ( $x - x_1, y - y_1$ ) are collinear, show that  $xy_1 = x_1y$ . Also, show that the line joining the given points passes through the origin. [HOTS]

5. Mr. Aggarwal starts walking from his home to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Mr. Aggarwal in reaching his office ? Assume that all distances covered are in straight lines. If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and

office at (13, 26) and coordinates are in km.

[HOTS]

6. Find the centre of a circle passing through the points (6, -6), (3, -7) and (3, 3).  
7. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3, 4), find its centroid.

## FORMATIVE ASSESSMENT

### Activity-1

**Objective :** To verify the distance formula and section formula. Or to verify the following :

- (i) The distance between the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- (ii) The coordinates of the point P, which divides the line segment joining the points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  in the ratio  $m : n$  are  $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$ .

- (iii) The coordinates of the mid-points P of the line segment joining the points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  are  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ .

**Materials Required :** Graph paper/squared paper, geometry box, etc.

**Procedure :**

- Take a 1 cm squared paper and on it draw the coordinate axes  $XOX'$  and  $YOY'$ .
- Plot the points  $A(-3, 1)$ ,  $B(5, 1)$  and  $C(1, 8)$  on the squared paper and join AB, BC and AC to get a  $\triangle ABC$ .
- Now using a pair of compasses, find the mid-points of AC and BC. Mark these mid-points as P and Q respectively.
- Mark the point R on AB such that  $AR = \frac{1}{4} AB$  or R divides AB in the ratio 1 : 3. Similarly, mark S on AB such that  $AS = \frac{3}{4} AB$  or S divides AB in the ratio 3 : 1.
- From the graph paper, write the coordinates of P, Q, R and S.

| Coordinates of |           |
|----------------|-----------|
| P              | (-1, 9/2) |
| Q              | (3, 9/2)  |
| R              | (-1, 1)   |
| S              | (3, 1)    |

**Observations :**

- Using a ruler, measure the length of AB, BC and CA

We have  $AB = BC = CA = 8$  cm.

- Now, using formula

$$AB = \sqrt{(5+3)^2 + (1-1)^2} \text{ cm} = 8 \text{ cm}$$

$$\begin{aligned} BC &= \sqrt{(1-5)^2 + (8-1)^2} \text{ cm} \\ &= \sqrt{16 + 49} \text{ cm} = \sqrt{65} \text{ cm} = 8.06 \text{ cm} \\ &= 8 \text{ cm} \end{aligned}$$

$$\begin{aligned} CA &= \sqrt{(1+3)^2 + (8-1)^2} \text{ cm} \\ &= \sqrt{16 + 49} \text{ cm} = \sqrt{65} \text{ cm} = 8.06 \text{ cm} = 8 \text{ cm} \end{aligned}$$

We see that in both the cases, the length of each side comes out to be 8 cm.

- Since P and Q are mid-points of AC and BC respectively, therefore, using formula, the coordinates of P are  $\left(\frac{1-3}{2}, \frac{8+1}{2}\right)$  or  $\left(-1, \frac{9}{2}\right)$  the coordinate of Q are  $\left(\frac{1+5}{2}, \frac{8+1}{2}\right)$  or  $\left(3, \frac{9}{2}\right)$   
Also, from the table : the coordinates of P and Q are same as obtained above.

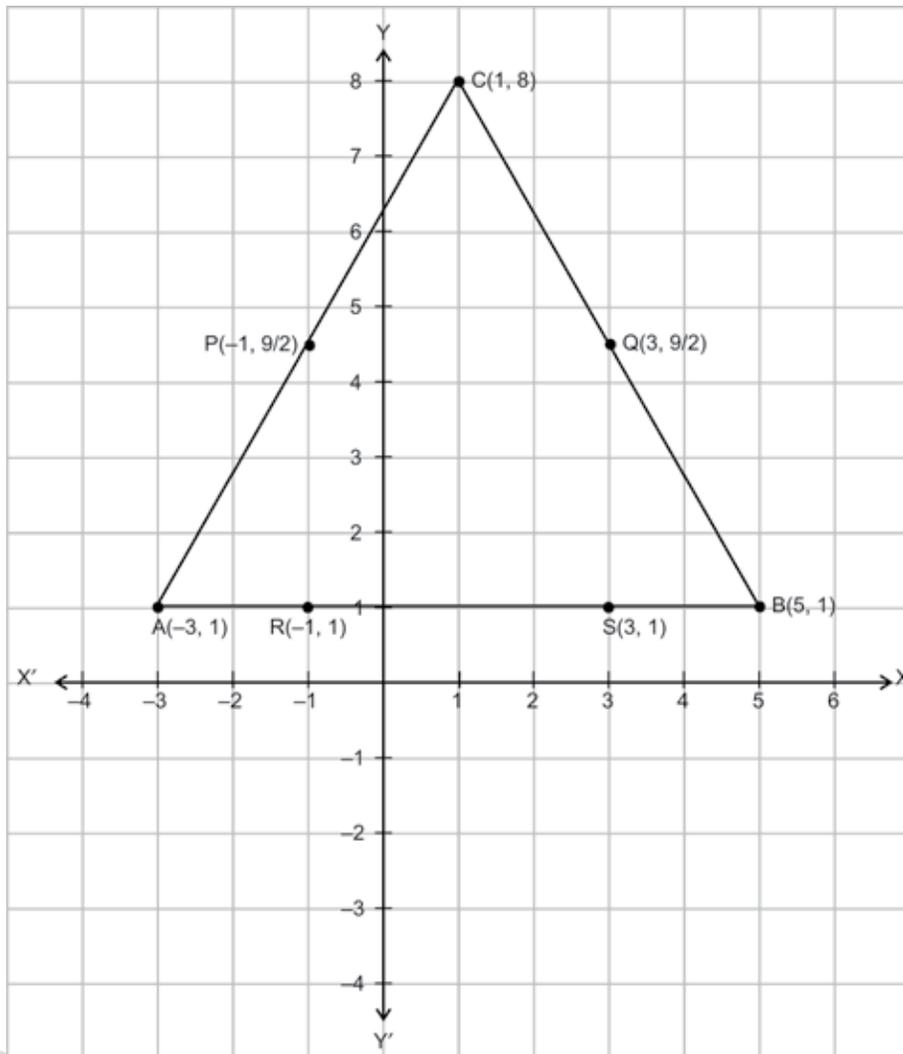
- Since R divides AB in the ratio 1 : 3, therefore,

coordinates of R (using formula) are  $\left(\frac{1 \times 5 + 3 \times (-3)}{1 + 3}, \frac{1 \times 1 + 3 \times 1}{1 + 3}\right)$  or  $(-1, 1)$ .

Similarly, S divides AB in the ratio 3 : 1, therefore, coordinates of S (using formula) are:

$\left(\frac{3 \times 5 + 1 \times (-3)}{3 + 1}, \frac{3 \times 1 + 1 \times 1}{3 + 1}\right)$  or  $(3, 1)$ .

Also, from the table, the coordinates of R and S are same as obtained above.



**Conclusion :** From the above activity it is verified that :

- (i) the distance of the line segments joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- (ii) If a point P divides the line segment joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the ratio  $m : n$ , then the coordinates of P are  $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$ .
- (iii) The coordinates of the mid-point of the line

segment joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$

are  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ .

**Do Yourself :** Draw three different triangles on a paper and in each case verify the above formulae.

### Activity-2

**Objective :** To verify the following formula :

The area of a triangle having vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by

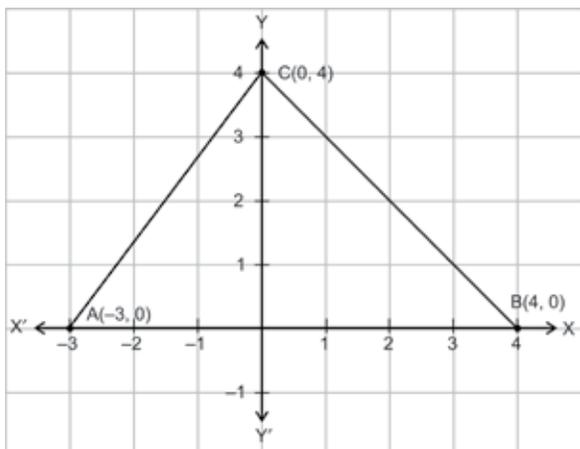
$$\frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

**Materials Required :** Graph paper/squared paper, geometry box etc.

**Procedure :** 1. Take a 1 cm squared paper and on it draw the coordinate axes  $XOX'$  and  $YOY'$ .

**Case I.**

- Plot the points  $A(-3, 0)$ ,  $B(4, 0)$  and  $C(0, 4)$  on the squared paper and join  $AB$ ,  $BC$  and  $AC$  to get a scalene triangle  $ABC$ .



- Find  $AB$ ,  $BC$  and  $AC$  using the distance formula.

$$\begin{aligned} AB &= \sqrt{(-3-4)^2 + (0-0)^2} \text{ cm} \\ &= \sqrt{49} \text{ cm} = 7 \text{ cm} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-0)^2 + (0-4)^2} \text{ cm} \\ &= \sqrt{16 + 16} \text{ cm} = 4\sqrt{2} \text{ cm} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-3-0)^2 + (0-4)^2} \text{ cm} \\ &= \sqrt{9 + 16} \text{ cm} = 5 \text{ cm} \end{aligned}$$

- Find the area of  $\Delta ABC$  using Heron's formula.

$$\text{Here, } a = 7, b = 4\sqrt{2}, c = 5$$

$$\therefore s = \frac{7 + 4\sqrt{2} + 5}{2} = 6 + 2\sqrt{2}$$

$\therefore$  Area of  $\Delta ABC$

$$\begin{aligned} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(6+2\sqrt{2})(6+2\sqrt{2}-7)(6+2\sqrt{2}-4\sqrt{2})(6+2\sqrt{2}-5)} \text{ cm}^2 \\ &= \sqrt{(6+2\sqrt{2})(2\sqrt{2}-1)(6-2\sqrt{2})(2\sqrt{2}+1)} \text{ cm}^2 \\ &= \sqrt{(6+2\sqrt{2})(6-2\sqrt{2})(2\sqrt{2}-1)(2\sqrt{2}+1)} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} &= \sqrt{(36-8)(8-1)} \text{ cm}^2 \\ &= \sqrt{28 \times 7} \text{ cm}^2 = 14 \text{ cm}^2 \end{aligned}$$

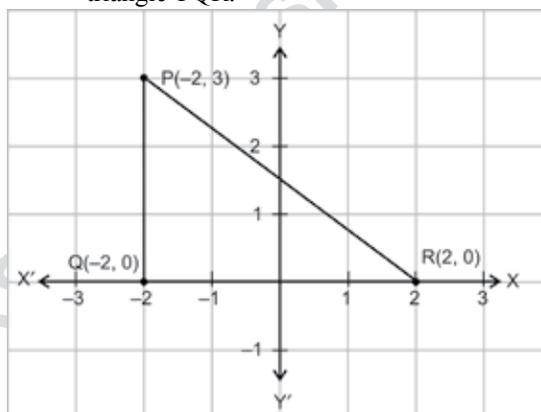
- Now, find the area of the triangle  $ABC$ , using the formula

$$\begin{aligned} &\frac{1}{2} [x_1(y_2 - y_3) - x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ \text{Area of } \Delta ABC &= \frac{1}{2} [-3(0-4) + 4(4-0) \\ &+ 0(0-0)] \text{ cm}^2 \\ &= \frac{1}{2} [12 + 16] \text{ cm}^2 = 14 \text{ cm}^2. \end{aligned}$$

**Observations :** From (3) and (4), we see that the area of  $\Delta ABC$  comes out to be same on both the cases.

**Case II.**

- Plot the points  $P(-2, 3)$ ,  $Q(-2, 0)$  and  $R(2, 0)$  and join  $PQ$ ,  $QR$  and  $PR$  to get the right triangle  $PQR$ .



- Find  $PQ$ ,  $QR$  and  $PR$  using the distance formula.

$$\begin{aligned} PQ &= \sqrt{(-2-2)^2 + (3-0)^2} \text{ cm} \\ &= \sqrt{9} \text{ cm} = 3 \text{ cm} \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(-2-2)^2 + (0-0)^2} \text{ cm} \\ &= \sqrt{16} \text{ cm} = 4 \text{ cm} \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(-2-2)^2 + (3-0)^2} \text{ cm} \\ &= \sqrt{16 + 9} \text{ cm} = 5 \text{ cm} \end{aligned}$$

- Find the area of  $\Delta PQR$  using Heron's formula:

$$\text{Here, } a = 3 \text{ cm, } b = 4 \text{ cm, } c = 5 \text{ cm}$$

$$\therefore s = \frac{3+4+5}{2} \text{ cm} = 6 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of } \Delta PQR &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{6(6-3)(6-4)(6-5)} \text{ cm}^2 \\ &= \sqrt{6 \times 3 \times 2 \times 1} \text{ cm}^2 = 6 \text{ cm}^2 \end{aligned}$$

4. Now, find the area of  $\Delta PQR$  using the formula

$$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\therefore \text{Area of } \Delta PQR = \frac{1}{2} [-2(0 - 0) - 2(0 - 3) + 2(3 - 0)] \text{ cm}^2 = \frac{1}{2} [6 + 6] = 6 \text{ cm}^2$$

**Observations :** From (3) and (4) above, we see that area of  $\Delta PQR$  comes out to be same in both the cases.

**Conclusion :** From the above activity it is verified that the area of a triangle having vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by

$$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

### Investigation

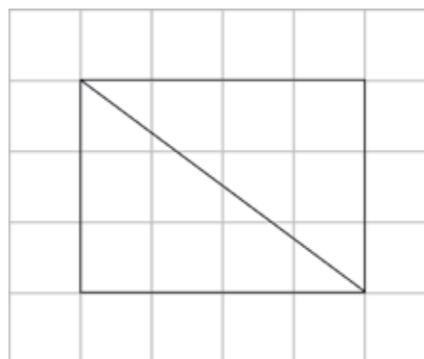
On a squared / graph paper, draw a rectangle of dimensions  $4 \times 3$ . Draw one of the diagonals of the rectangle. The diagonal passes through 6 squares.

Now on a squared paper draw several rectangles of different sizes. The length ( $l$ ) and breadth ( $b$ ) of each rectangle must be a whole number of squares with a common factor of 1 only. For example, sides 4 squares by 9 squares is acceptable but 4 squares by 6 squares is not because 4 and 6 have a common factor 2.

For each rectangle, draw a diagonal and count the number of squares through which the diagonal passes.

The dimensions of a rectangle is  $120 \times 91$ . How many squares will the diagonal pass through?

If the dimensions of a rectangle are  $m \times n$ , where  $m$  and  $n$  are co-primes, then how many squares will the diagonal pass through?



### Puzzle

A merchant has nine gold coins which look identical but in fact one of the coins is an underweight fake. Investigate how the merchant can use only a balance to find the fake coin in just two weighings.

**Exercise 7.1****Question 1:**

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) (ii) (-5, 7), (-1, 3) (iii) (a, b), (-a, -b)

Answer:

(i) Distance between the two points is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore, distance between (2, 3) and (4, 1) is given by

$$\begin{aligned} l &= \sqrt{(2-4)^2 + (3-1)^2} = \sqrt{(-2)^2 + (2)^2} \\ &= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \end{aligned}$$

(ii) Distance between (-5, 7) and (-1, 3) is given by

$$\begin{aligned} l &= \sqrt{(-5-(-1))^2 + (7-3)^2} = \sqrt{(-4)^2 + (4)^2} \\ &= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \end{aligned}$$

(iii) Distance between (a, b) and (-a, -b) is given by

$$\begin{aligned} l &= \sqrt{(a-(-a))^2 + (b-(-b))^2} \\ &= \sqrt{(2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \end{aligned}$$

**Question 2:**

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer:

Distance between points (0,0) and (36,15)



$$\begin{aligned} &= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2} \\ &= \sqrt{1296 + 225} = \sqrt{1521} = 39 \end{aligned}$$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A.

And hence, as calculated above, the distance between town A and B will be 39 km.

**Question 3:**

Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Answer:

Let the points (1, 5), (2, 3), and (-2, -11) be representing the vertices A, B, and C of the given triangle respectively.

Let  $A = (1, 5), B = (2, 3), C = (-2, -11)$

$$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$$

$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212}$$

$$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9 + 256} = \sqrt{265}$$

Since  $AB + BC \neq CA$ ,

Therefore, the points (1, 5), (2, 3), and (-2, -11) are not collinear.

**Question 4:**

Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Answer:

Let the points (5, -2), (6, 4), and (7, -2) are representing the vertices A, B, and C of the given triangle respectively.



$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

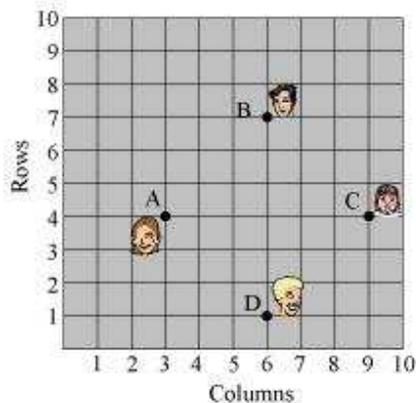
Therefore,  $AB = BC$

As two sides are equal in length, therefore, ABC is an isosceles triangle.

**Question 5:**

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.

Using distance formula, find which of them is correct.



Answer:

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.



$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

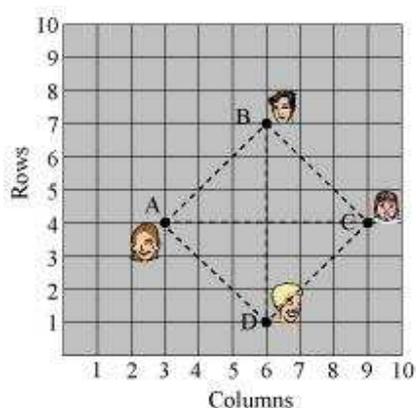
$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CB = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal BD} = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

#### Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i)  $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

(ii)  $(-3, 5), (3, 1), (0, 3), (-1, -4)$

(iii)  $(4, 5), (7, 6), (4, 3), (1, 2)$



Answer:

(i) Let the points  $(-1, -2)$ ,  $(1, 0)$ ,  $(-1, 2)$ , and  $(-3, 0)$  be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal BD} = \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii) Let the points  $(-3, 5)$ ,  $(3, 1)$ ,  $(0, 3)$ , and  $(-1, -4)$  be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} = \sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths.

Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii) Let the points  $(4, 5)$ ,  $(7, 6)$ ,  $(4, 3)$ , and  $(1, 2)$  be representing the vertices A, B, C, and D of the given quadrilateral respectively.



$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$

$$\text{Diagonal BD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 13\sqrt{2}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

#### Question 7:

Find the point on the x-axis which is equidistant from  $(2, -5)$  and  $(-2, 9)$ .

Answer:

We have to find a point on x-axis. Therefore, its y-coordinate will be 0.

Let the point on x-axis be  $(x, 0)$ .

$$\text{Distance between } (x, 0) \text{ and } (2, -5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$

$$\text{Distance between } (x, 0) \text{ and } (-2, 9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (9)^2}$$

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$8x = 25 - 81$$

$$8x = -56$$

$$x = -7$$



Therefore, the point is  $(-7, 0)$ .

**Question 8:**

Find the values of  $y$  for which the distance between the points  $P(2, -3)$  and  $Q(10, y)$  is 10 units.

Answer:

It is given that the distance between  $(2, -3)$  and  $(10, y)$  is 10.

$$\text{Therefore, } \sqrt{(2-10)^2 + (-3-y)^2} = 10$$

$$\sqrt{(-8)^2 + (3+y)^2} = 10$$

$$64 + (y+3)^2 = 100$$

$$(y+3)^2 = 36$$

$$y+3 = \pm 6$$

$$y+3 = 6 \text{ or } y+3 = -6$$

$$\text{Therefore, } y = 3 \text{ or } -9$$

**Question 9:**

If  $Q(0, 1)$  is equidistant from  $P(5, -3)$  and  $R(x, 6)$ , find the values of  $x$ . Also find the distance  $QR$  and  $PR$ .

Answer:

$$PQ = QR$$

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2+25}$$

$$41 = x^2 + 25$$

$$16 = x^2$$

$$x = \pm 4$$

Therefore, point  $R$  is  $(4, 6)$  or  $(-4, 6)$ .



When point R is (4, 6),

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$$

$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (-4, 6),

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

**Question 10:**

Find a relation between  $x$  and  $y$  such that the point  $(x, y)$  is equidistant from the point  $(3, 6)$  and  $(-3, 4)$ .

Answer:

Point  $(x, y)$  is equidistant from  $(3, 6)$  and  $(-3, 4)$ .

$$\therefore \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$36 - 16 = 6x + 6x + 12y - 8y$$

$$20 = 12x + 4y$$

$$3x + y = 5$$

$$3x + y - 5 = 0$$

**Exercise 7.2****Question 1:**

Find the coordinates of the point which divides the join of  $(-1, 7)$  and  $(4, -3)$  in the ratio 2:3.

Answer:

Let  $P(x, y)$  be the required point. Using the section formula, we obtain

$$x = \frac{2 \times 4 + 3 \times (-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

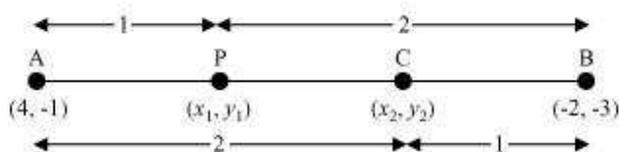
$$y = \frac{2 \times (-3) + 3 \times 7}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Therefore, the point is  $(1, 3)$ .

**Question 2:**

Find the coordinates of the points of trisection of the line segment joining  $(4, -1)$  and  $(-2, -3)$ .

Answer:



Let  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  are the points of trisection of the line segment joining the given points i.e.,  $AP = PQ = QB$

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1 \times (-2) + 2 \times 4}{1 + 2}, \quad y_1 = \frac{1 \times (-3) + 2 \times (-1)}{1 + 2}$$

$$x_1 = \frac{-2 + 8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3 - 2}{3} = \frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1) = \left(2, -\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.



$$x_2 = \frac{2 \times (-2) + 1 \times 4}{2+1}, \quad y_2 = \frac{2 \times (-3) + 1 \times (-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \quad y_2 = \frac{-6-1}{3} = -\frac{7}{3}$$

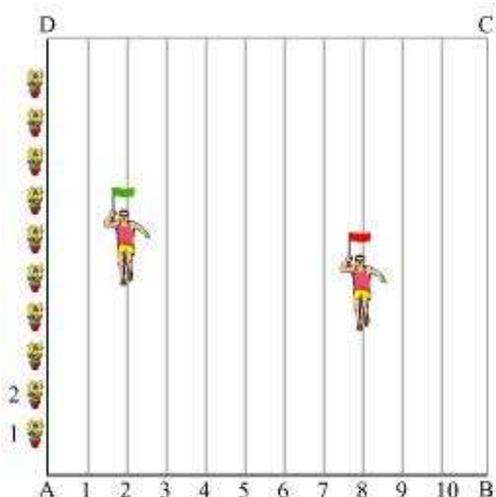
$$Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

**Question 3:**

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the

following figure. Niharika runs  $\frac{1}{4}$ th the distance AD on the 2<sup>nd</sup> line and posts a green

flag. Preet runs  $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?





Answer:

It can be observed that Niharika posted the green flag at  $\frac{1}{4}$  of the distance AD i.e.,

$\left(\frac{1}{4} \times 100\right) \text{ m} = 25$  m from the starting point of 2<sup>nd</sup> line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at  $\frac{1}{5}$  of the distance AD i.e.,  $\left(\frac{1}{5} \times 100\right) \text{ m} = 20$  m from the starting point of 8<sup>th</sup> line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61} \text{ m}$$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x, y).

$$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5, \quad y = \frac{45}{2} = 22.5$$

$$\text{Hence, } A(x, y) = (5, 22.5)$$

Therefore, Rashmi should post her blue flag at 22.5m on 5<sup>th</sup> line

**Question 4:**

Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Answer:

Let the ratio in which the line segment joining (–3, 10) and (6, –8) is divided by point (–1, 6) be k : 1.



$$\text{Therefore, } -1 = \frac{6k-3}{k+1}$$

$$-k-1 = 6k-3$$

$$7k = 2$$

$$k = \frac{2}{7}$$

Therefore, the required ratio is 2 : 7.

**Question 5:**

Find the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer:

Let the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by x-axis be  $k:1$ .

Therefore, the coordinates of the point of division is  $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$ .

We know that y-coordinate of any point on x-axis is 0.

$$\therefore \frac{5k-5}{k+1} = 0$$

$$k = 1$$

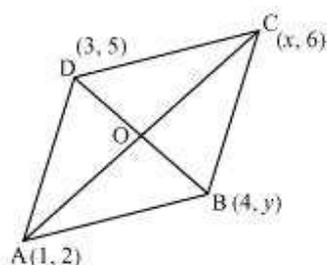
Therefore, x-axis divides it in the ratio 1:1.

$$\text{Division point} = \left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right) = \left(\frac{-4+1}{2}, \frac{5-5}{2}\right) = \left(\frac{-3}{2}, 0\right)$$

**Question 6:**

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer:



Let  $(1, 2)$ ,  $(4, y)$ ,  $(x, 6)$ , and  $(3, 5)$  are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2} = \frac{7}{2} \text{ and } 4 = \frac{5+y}{2}$$

$$\Rightarrow x+1 = 7 \text{ and } 5+y = 8$$

$$\Rightarrow x = 6 \text{ and } y = 3$$

#### Question 7:

Find the coordinates of a point A, where AB is the diameter of circle whose centre is  $(2, -3)$  and B is  $(1, 4)$

Answer:

Let the coordinates of point A be  $(x, y)$ .

Mid-point of AB is  $(2, -3)$ , which is the center of the circle.



$$\therefore (2, -3) = \left( \frac{x+1}{2}, \frac{y+4}{2} \right)$$

$$\Rightarrow \frac{x+1}{2} = 2 \text{ and } \frac{y+4}{2} = -3$$

$$\Rightarrow x+1 = 4 \text{ and } y+4 = -6$$

$$\Rightarrow x = 3 \text{ and } y = -10$$

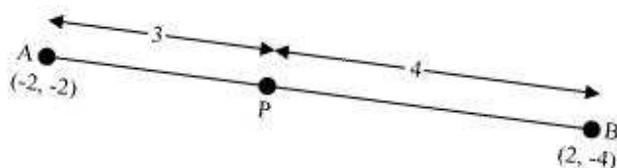
Therefore, the coordinates of A are  $(3, -10)$ .

**Question 8:**

If A and B are  $(-2, -2)$  and  $(2, -4)$ , respectively, find the coordinates of P such

that  $AP = \frac{3}{7} AB$  and P lies on the line segment AB.

Answer:



The coordinates of point A and B are  $(-2, -2)$  and  $(2, -4)$  respectively.

Since  $AP = \frac{3}{7} AB$ ,

Therefore,  $AP: PB = 3:4$

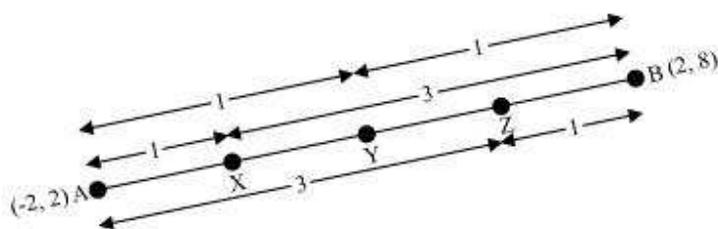
Point P divides the line segment AB in the ratio 3:4.

$$\begin{aligned} \text{Coordinates of P} &= \left( \frac{3 \times 2 + 4 \times (-2)}{3+4}, \frac{3 \times (-4) + 4 \times (-2)}{3+4} \right) \\ &= \left( \frac{6-8}{7}, \frac{-12-8}{7} \right) \\ &= \left( -\frac{2}{7}, -\frac{20}{7} \right) \end{aligned}$$

**Question 9:**

Find the coordinates of the points which divide the line segment joining A (– 2, 2) and B (2, 8) into four equal parts.

Answer:



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\begin{aligned}\text{Coordinates of P} &= \left( \frac{1 \times 2 + 3 \times (-2)}{1+3}, \frac{1 \times 8 + 3 \times 2}{1+3} \right) \\ &= \left( -1, \frac{7}{2} \right)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of Q} &= \left( \frac{2 + (-2)}{2}, \frac{2 + 8}{2} \right) \\ &= (0, 5)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of R} &= \left( \frac{3 \times 2 + 1 \times (-2)}{3+1}, \frac{3 \times 8 + 1 \times 2}{3+1} \right) \\ &= \left( 1, \frac{13}{2} \right)\end{aligned}$$

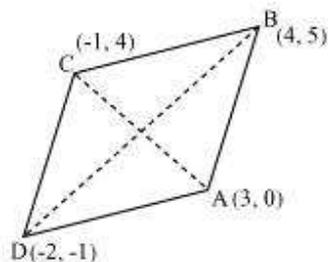
**Question 10:**

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1)

taken in order. [**Hint:** Area of a rhombus =  $\frac{1}{2}$  (product of its diagonals)]



Answer:



Let  $(3, 0)$ ,  $(4, 5)$ ,  $(-1, 4)$  and  $(-2, -1)$  are the vertices A, B, C, D of a rhombus ABCD.

$$\begin{aligned}\text{Length of diagonal AC} &= \sqrt{[3 - (-1)]^2 + (0 - 4)^2} \\ &= \sqrt{16 + 16} = 4\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Length of diagonal BD} &= \sqrt{[4 - (-2)]^2 + [5 - (-1)]^2} \\ &= \sqrt{36 + 36} = 6\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Therefore, area of rhombus ABCD} &= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \\ &= 24 \text{ square units}\end{aligned}$$

**Exercise 7.3****Question 1:**

Find the area of the triangle whose vertices are:

(i) (2, 3), (-1, 0), (2, -4) (ii) (-5, -1), (3, -5), (5, 2)

Answer:

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of the given triangle} &= \frac{1}{2} [2\{0 - (-4)\} + (-1)\{(-4) - (3)\} + 2(3 - 0)] \\ &= \frac{1}{2} \{8 + 7 + 6\} \\ &= \frac{21}{2} \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{Area of the given triangle} &= \frac{1}{2} [(-5)\{(-5) - (-2)\} + 3(2 - (-1)) + 5\{-1 - (-5)\}] \\ \text{(ii)} & \\ &= \frac{1}{2} \{35 + 9 + 20\} \\ &= 32 \text{ square units} \end{aligned}$$

**Question 2:**

In each of the following find the value of 'k', for which the points are collinear.

(i) (7, -2), (5, 1), (3, -k) (ii) (8, 1), (k, -4), (2, -5)

Answer:

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, -2) (5, 1), and (3, k), area = 0



$$\frac{1}{2} [7\{1-k\} + 5\{k-(-2)\} + 3\{(-2)-1\}] = 0$$

$$7 - 7k + 5k + 10 - 9 = 0$$

$$-2k + 8 = 0$$

$$k = 4$$

(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points  $(8, 1)$ ,  $(k, -4)$ , and  $(2, -5)$ , area = 0

$$\frac{1}{2} [8\{-4-(-5)\} + k\{(-5)-(1)\} + 2\{1-(-4)\}] = 0$$

$$8 - 6k + 10 = 0$$

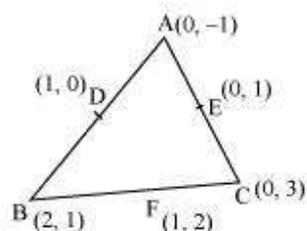
$$6k = 18$$

$$k = 3$$

### Question 3:

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are  $(0, -1)$ ,  $(2, 1)$  and  $(0, 3)$ . Find the ratio of this area to the area of the given triangle.

Answer:



Let the vertices of the triangle be  $A(0, -1)$ ,  $B(2, 1)$ ,  $C(0, 3)$ .

Let  $D, E, F$  be the mid-points of the sides of this triangle. Coordinates of  $D, E,$  and  $F$  are given by



$$D = \left( \frac{0+2}{2}, \frac{-1+1}{2} \right) = (1, 0)$$

$$E = \left( \frac{0+0}{2}, \frac{3-1}{2} \right) = (0, 1)$$

$$F = \left( \frac{2+0}{2}, \frac{1+3}{2} \right) = (1, 2)$$

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle DEF &= \frac{1}{2} \{1(2-1) + 1(1-0) + 0(0-2)\} \\ &= \frac{1}{2} (1+1) = 1 \text{ square units} \end{aligned}$$

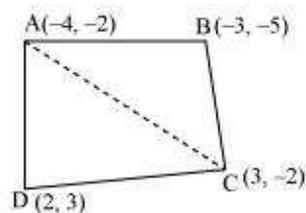
$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [0(1-3) + 2\{3-(-1)\} + 0(-1-1)] \\ &= \frac{1}{2} \{8\} = 4 \text{ square units} \end{aligned}$$

Therefore, required ratio = 1 : 4

#### Question 4:

Find the area of the quadrilateral whose vertices, taken in order, are  $(-4, -2)$ ,  $(-3, -5)$ ,  $(3, -2)$  and  $(2, 3)$

Answer:



Let the vertices of the quadrilateral be A  $(-4, -2)$ , B  $(-3, -5)$ , C  $(3, -2)$ , and D  $(2, 3)$ . Join AC to form two triangles  $\triangle ABC$  and  $\triangle ADC$ .



$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [(-4)\{(-5) - (-2)\} + (-3)\{(-2) - (-2)\} + 3\{(-2) - (-5)\}] \\ &= \frac{1}{2} (12 + 0 + 9) = \frac{21}{2} \text{ square units} \end{aligned}$$

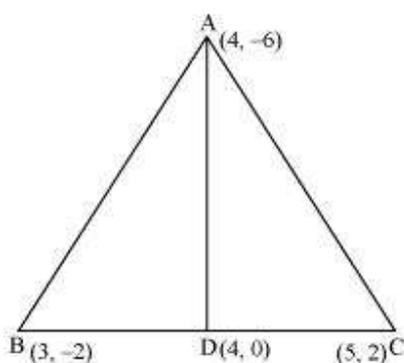
$$\begin{aligned} \text{Area of } \triangle ACD &= \frac{1}{2} [(-4)\{(-2) - (3)\} + 3\{(3) - (-2)\} + 2\{(-2) - (-2)\}] \\ &= \frac{1}{2} \{20 + 15 + 0\} = \frac{35}{2} \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{Area of } \square ABCD &= \text{Area of } \triangle ABC + \text{Area of } \triangle ACD \\ &= \left(\frac{21}{2} + \frac{35}{2}\right) \text{ square units} = 28 \text{ square units} \end{aligned}$$

**Question 5:**

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for  $\triangle ABC$  whose vertices are A (4, -6), B (3, -2) and C (5, 2)

Answer:



Let the vertices of the triangle be A (4, -6), B (3, -2), and C (5, 2).

Let D be the mid-point of side BC of  $\triangle ABC$ . Therefore, AD is the median in  $\triangle ABC$ .

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$$



$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\begin{aligned} \text{Area of } \triangle ABD &= \frac{1}{2} [(4)\{-2\} - (0)\{0\} + (3)\{0 - (-6)\} + (4)\{-6 - (-2)\}] \\ &= \frac{1}{2} (-8 + 18 - 16) = -3 \text{ square units} \end{aligned}$$

However, area cannot be negative. Therefore, area of  $\triangle ABD$  is 3 square units.

$$\begin{aligned} \text{Area of } \triangle ADC &= \frac{1}{2} [(4)\{0 - (2)\} + (4)\{(2) - (-6)\} + (5)\{-6 - (0)\}] \\ &= \frac{1}{2} (-8 + 32 - 30) = -3 \text{ square units} \end{aligned}$$

However, area cannot be negative. Therefore, area of  $\triangle ADC$  is 3 square units.

Clearly, median AD has divided  $\triangle ABC$  in two triangles of equal areas.

**Exercise 7.4****Question 1:**

Determine the ratio in which the line  $2x + y - 4 = 0$  divides the line segment joining the points  $A(2, -2)$  and  $B(3, 7)$

Answer:

Let the given line divide the line segment joining the points  $A(2, -2)$  and  $B(3, 7)$  in a ratio  $k : 1$ .

Coordinates of the point of division  $= \left( \frac{3k+2}{k+1}, \frac{7k-2}{k+1} \right)$

This point also lies on  $2x + y - 4 = 0$

$$\therefore 2 \left( \frac{3k+2}{k+1} \right) + \left( \frac{7k-2}{k+1} \right) - 4 = 0$$

$$\Rightarrow \frac{6k+4+7k-2-4k-4}{k+1} = 0$$

$$\Rightarrow 9k - 2 = 0$$

$$\Rightarrow k = \frac{2}{9}$$

Therefore, the ratio in which the line  $2x + y - 4 = 0$  divides the line segment joining the points  $A(2, -2)$  and  $B(3, 7)$  is 2:9.

**Question 2:**

Find a relation between  $x$  and  $y$  if the points  $(x, y)$ ,  $(1, 2)$  and  $(7, 0)$  are collinear.

Answer:

If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$



$$\text{Area} = \frac{1}{2} [x(2-0) + 1(0-y) + 7(y-2)]$$

$$0 = \frac{1}{2} [2x - y + 7y - 14]$$

$$0 = \frac{1}{2} [2x + 6y - 14]$$

$$2x + 6y - 14 = 0$$

$$x + 3y - 7 = 0$$

This is the required relation between  $x$  and  $y$ .

**Question 3:**

Find the centre of a circle passing through the points  $(6, -6)$ ,  $(3, -7)$  and  $(3, 3)$ .

Answer:

Let  $O(x, y)$  be the centre of the circle. And let the points  $(6, -6)$ ,  $(3, -7)$ , and  $(3, 3)$  be representing the points  $A$ ,  $B$ , and  $C$  on the circumference of the circle.

$$\therefore OA = \sqrt{(x-6)^2 + (y+6)^2}$$

$$OB = \sqrt{(x-3)^2 + (y+7)^2}$$

$$OC = \sqrt{(x-3)^2 + (y-3)^2}$$

However,  $OA = OB$  (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$$

$$\Rightarrow -6x - 2y + 14 = 0$$

$$\Rightarrow 3x + y = 7 \quad \dots (1)$$

Similarly,  $OA = OC$  (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$$

$$\Rightarrow -6x + 18y + 54 = 0$$

$$\Rightarrow -3x + 9y = -27 \quad \dots (2)$$



On adding equation (1) and (2), we obtain

$$10y = -20$$

$$y = -2$$

From equation (1), we obtain

$$3x - 2 = 7$$

$$3x = 9$$

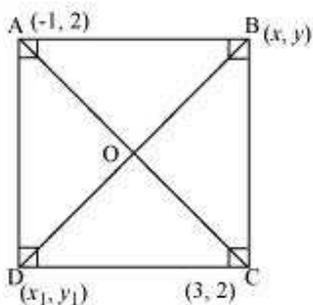
$$x = 3$$

Therefore, the centre of the circle is  $(3, -2)$ .

**Question 4:**

The two opposite vertices of a square are  $(-1, 2)$  and  $(3, 2)$ . Find the coordinates of the other two vertices.

Answer:



Let ABCD be a square having  $(-1, 2)$  and  $(3, 2)$  as vertices A and C respectively. Let  $(x, y)$ ,  $(x_1, y_1)$  be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

$$\therefore AB = BC$$

$$\Rightarrow \sqrt{(x+1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y-2)^2}$$

$$\Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y$$

$$\Rightarrow 8x = 8$$

$$\Rightarrow x = 1$$

We know that in a square, all interior angles are of  $90^\circ$ .



In  $\triangle ABC$ ,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \left( \sqrt{(1+1)^2 + (y-2)^2} \right)^2 + \left( \sqrt{(1-3)^2 + (y-2)^2} \right)^2 = \left( \sqrt{(3+1)^2 + (2-2)^2} \right)^2$$

$$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$$

$$\Rightarrow 2y^2 + 16 - 8y = 16$$

$$\Rightarrow 2y^2 - 8y = 0$$

$$\Rightarrow y(y - 4) = 0$$

$$\Rightarrow y = 0 \text{ or } 4$$

We know that in a square, the diagonals are of equal length and bisect each other at  $90^\circ$ . Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinate of point O} = \left( \frac{-1+3}{2}, \frac{2+2}{2} \right)$$

$$\left( \frac{1+x_1}{2}, \frac{y+y_1}{2} \right) = (1, 2)$$

$$\frac{1+x_1}{2} = 1$$

$$1+x_1 = 2$$

$$x_1 = 1$$

$$\text{and } \frac{y+y_1}{2} = 2$$

$$\Rightarrow y + y_1 = 4$$

$$\text{If } y = 0,$$

$$y_1 = 4$$

$$\text{If } y = 4,$$

$$y_1 = 0$$

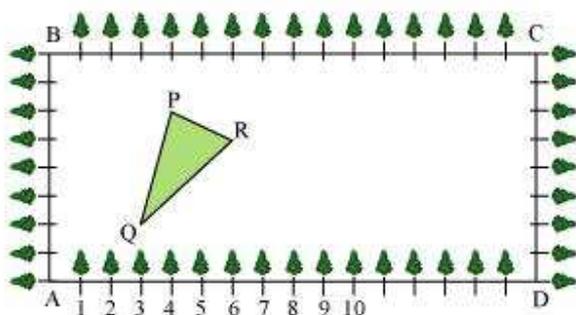
Therefore, the required coordinates are (1, 0) and (1, 4).

#### Question 5:

The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted



on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of  $\Delta PQR$  if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

Answer:

(i) Taking A as origin, we will take AD as x-axis and AB as y-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\begin{aligned}\text{Area of triangle PQR} &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [4(2 - 5) + 3(5 - 6) + 6(6 - 2)] \\ &= \frac{1}{2} [-12 - 3 + 24] \\ &= \frac{9}{2} \text{ square units}\end{aligned}$$

(ii) Taking C as origin, CB as x-axis, and CD as y-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.



$$\begin{aligned}\text{Area of triangle PQR} &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [12(6 - 3) + 13(3 - 2) + 10(2 - 6)] \\ &= \frac{1}{2} [36 + 13 - 40] \\ &= \frac{9}{2} \text{ square units}\end{aligned}$$

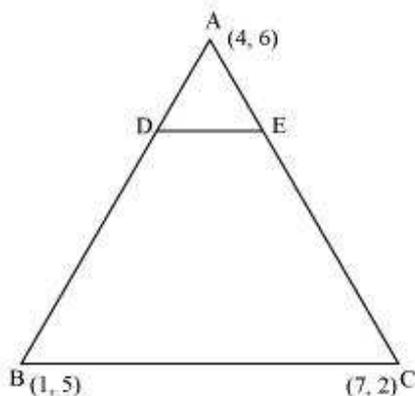
It can be observed that the area of the triangle is same in both the cases.

**Question 6:**

The vertices of a  $\triangle ABC$  are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect

sides AB and AC at D and E respectively, such that  $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$ . Calculate the area of the  $\triangle ADE$  and compare it with the area of  $\triangle ABC$ . (Recall Converse of basic proportionality theorem and Theorem 6.6 related to ratio of areas of two similar triangles)

Answer:



Given that,  $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$



$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC} = \frac{1}{4}$$
$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\text{Coordinates of Point D} = \left( \frac{1 \times 1 + 3 \times 4}{1+3}, \frac{1 \times 5 + 3 \times 6}{1+3} \right)$$
$$= \left( \frac{13}{4}, \frac{23}{4} \right)$$

$$\text{Coordinates of point E} = \left( \frac{1 \times 7 + 3 \times 4}{1+3}, \frac{1 \times 2 + 3 \times 6}{1+3} \right)$$
$$= \left( \frac{19}{4}, \frac{20}{4} \right)$$

$$\text{Area of a triangle} = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\text{Area of } \triangle ADE = \frac{1}{2} \left[ 4 \left( \frac{23}{4} - \frac{20}{4} \right) + \frac{13}{4} \left( \frac{20}{4} - 6 \right) + \frac{19}{4} \left( 6 - \frac{23}{4} \right) \right]$$
$$= \frac{1}{2} \left[ 3 - \frac{13}{4} + \frac{19}{16} \right] = \frac{1}{2} \left[ \frac{48 - 52 + 19}{16} \right] = \frac{15}{32} \text{ square units}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} [4(5-2) + 1(2-6) + 7(6-5)]$$
$$= \frac{1}{2} [12 - 4 + 7] = \frac{15}{2} \text{ square units}$$

Clearly, the ratio between the areas of  $\triangle ADE$  and  $\triangle ABC$  is 1:16.

**Alternatively,**

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here  $\triangle ADE$  and  $\triangle ABC$ ) will be similar to each other.



Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

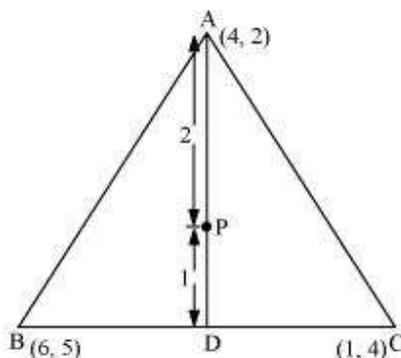
Therefore, ratio between the areas of  $\triangle ADE$  and  $\triangle ABC = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

**Question 7:**

Let  $A(4, 2)$ ,  $B(6, 5)$  and  $C(1, 4)$  be the vertices of  $\triangle ABC$ .

- The median from  $A$  meets  $BC$  at  $D$ . Find the coordinates of point  $D$ .
- Find the coordinates of the point  $P$  on  $AD$  such that  $AP:PD = 2:1$
- Find the coordinates of point  $Q$  and  $R$  on medians  $BE$  and  $CF$  respectively such that  $BQ:QE = 2:1$  and  $CR:RF = 2:1$ .
- What do you observe?
- If  $A(x_1, y_1)$ ,  $B(x_2, y_2)$ , and  $C(x_3, y_3)$  are the vertices of  $\triangle ABC$ , find the coordinates of the centroid of the triangle.

Answer:



- Median  $AD$  of the triangle will divide the side  $BC$  in two equal parts. Therefore,  $D$  is the mid-point of side  $BC$ .

$$\text{Coordinates of } D = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

- Point  $P$  divides the side  $AD$  in a ratio  $2:1$ .



$$\text{Coordinates of P} = \left( \frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}, \frac{2 \times \frac{9}{2} + 1 \times 2}{2+1} \right) = \left( \frac{11}{3}, \frac{11}{3} \right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left( \frac{4+1}{2}, \frac{2+4}{2} \right) = \left( \frac{5}{2}, 3 \right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left( \frac{2 \times \frac{5}{2} + 1 \times 6}{2+1}, \frac{2 \times 3 + 1 \times 5}{2+1} \right) = \left( \frac{11}{3}, \frac{11}{3} \right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left( \frac{4+6}{2}, \frac{2+5}{2} \right) = \left( 5, \frac{7}{2} \right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left( \frac{2 \times 5 + 1 \times 1}{2+1}, \frac{2 \times \frac{7}{2} + 1 \times 4}{2+1} \right) = \left( \frac{11}{3}, \frac{11}{3} \right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle,  $\Delta ABC$ , having its vertices as  $A(x_1, y_1)$ ,  $B(x_2, y_2)$ , and  $C(x_3, y_3)$ .

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.



$$\text{Coordinates of } D = \left( \frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right)$$

Let the centroid of this triangle be O.

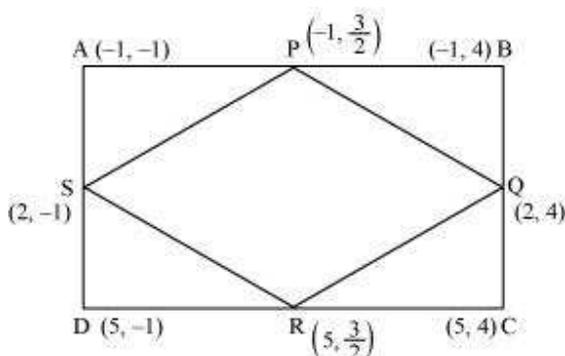
Point O divides the side AD in a ratio 2:1.

$$\begin{aligned} \text{Coordinates of } O &= \left( \frac{2 \times \frac{x_2 + x_3}{2} + 1 \times x_1}{2+1}, \frac{2 \times \frac{y_2 + y_3}{2} + 1 \times y_1}{2+1} \right) \\ &= \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \end{aligned}$$

#### Question 8:

ABCD is a rectangle formed by the points A (– 1, – 1), B (– 1, 4), C (5, 4) and D (5, – 1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Answer:



P is the mid-point of side AB.

$$\text{Therefore, the coordinates of P are } \left( \frac{-1-1}{2}, \frac{-1+4}{2} \right) = \left( -1, \frac{3}{2} \right)$$

Similarly, the coordinates of Q, R, and S are (2, 4),  $\left( 5, \frac{3}{2} \right)$ , and (2, – 1) respectively.



$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$$

$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.