Question 8.1:

Answer the following:

You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

Answer

Answer: (a) No (b) Yes

Gravitational influence of matter on nearby objects cannot be screened by any means. This is because gravitational force unlike electrical forces is independent of the nature of the material medium. Also, it is independent of the status of other objects.

If the size of the space station is large enough, then the astronaut will detect the change in Earth's gravity (g).

Tidal effect depends inversely upon the cube of the distance while, gravitational force depends inversely on the square of the distance. Since the distance between the Moon and the Earth is smaller than the distance between the Sun and the Earth, the tidal effect of the Moon's pull is greater than the tidal effect of the Sun's pull.

Question 8.2:

Choose the correct alternative:

Acceleration due to gravity increases/decreases with increasing altitude.

Acceleration due to gravity increases/decreases with increasing depth. (assume the earth to be a sphere of uniform density).

Acceleration due to gravity is independent of mass of the earth/mass of the body.

The formula $-G\,Mm(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

Answer

Answer:

Decreases

Decreases

Mass of the body

More

Explanation:

Acceleration due to gravity at depth h is given by the relation:

$$g_h = \left(1 - \frac{2h}{R_e}\right)g$$

Where,

R_e = Radius of the Earth

g = Acceleration due to gravity on the surface of the Earth

It is clear from the given relation that acceleration due to gravity decreases with an increase in height.

Acceleration due to gravity at depth d is given by the relation:

$$g_d = \left(1 - \frac{d}{R_e}\right)g$$

It is clear from the given relation that acceleration due to gravity decreases with an increase in depth.

Acceleration due to gravity of body of mass m is given by the relation:

$$g = \frac{GM}{R^2}$$

Where,

G = Universal gravitational constant

M = Mass of the Earth

R = Radius of the Earth

Hence, it can be inferred that acceleration due to gravity is independent of the mass of the body.

Gravitational potential energy of two points r_2 and r_1 distance away from the centre of the Earth is respectively given by:

$$V(r_1) = -\frac{GmM}{r_1}$$

$$V(r_2) = -\frac{GmM}{r_2}$$

$$\therefore \text{Difference in potential energy, } V = V(r_2) - V(r_1) = -GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Hence, this formula is more accurate than the formula $mg(r_2 - r_1)$.

Question 8.3:

Suppose there existed a planet that went around the sun twice as fast as the earth.What would be its orbital size as compared to that of the earth?

Answer

Answer: Lesser by a factor of 0.63

Time taken by the Earth to complete one revolution around the Sun,

T_e = 1 year

Orbital radius of the Earth in its orbit, R_e = 1 AU

Time taken by the planet to complete one revolution around the Sun, $T_P = \frac{1}{2}T_e = \frac{1}{2}$ year

Orbital radius of the planet = R_p

From Kepler's third law of planetary motion, we can write:

$$\left(\frac{R_P}{R_e}\right)^3 = \left(\frac{T_P}{T_e}\right)^2$$

$$\frac{R_P}{R_e} = \left(\frac{T_P}{T_e}\right)^{\frac{2}{3}}$$

$$= \left(\frac{\frac{1}{2}}{1}\right)^{\frac{2}{3}} = (0.5)^{\frac{2}{3}} = 0.63$$

Hence, the orbital radius of the planet will be 0.63 times smaller than that of the Earth.

Question 8.4:

Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

Answer

Orbital period of $I_0, T_{Io} = 1.769\,\text{days} = 1.769 \times 24 \times 60 \times 60$ s

Orbital radius of I_0, $R_{Io} = 4.22 \times 10^8$ m

Satellite I_0 is revolving around the Jupiter

Mass of the latter is given by the relation:

$$M_J = \frac{4\pi^2 R_{\pm}^3}{G T_{Io}^2} \qquad \dots (i)$$

Where,

M_J = Mass of Jupiter

G = Universal gravitational constant

Orbital period of the Earth,

$T_e = 365.25 \text{ days} = 365.25 \times 24 \times 60 \times 60 \text{ s}$

Orbital radius of the Earth,

$R_e = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$

Mass of Sun is given as:

$$M_s = \frac{4\pi^2 R_e^3}{G T_e^2} \qquad \dots (ii)$$

$$\therefore \frac{M_s}{M_J} = \frac{4\pi^2 R_e^3}{G T_e^2} \times \frac{G T_{Io}^2}{4\pi^2 R_{Io}^3} = \frac{R_e^3}{R_{Io}^3} \times \frac{T_{Io}^2}{T_e^2}$$

$$= \left(\frac{1.769 \times 24 \times 60 \times 60}{365.25 \times 24 \times 60 \times 60}\right)^2 \times \left(\frac{1.496 \times 10^{11}}{4.22 \times 10^8}\right)^3$$

$$= 1045.04$$

$$\therefore \frac{M_S}{M_J} \sim 1000$$

$$M_S \sim 1000 \times M_J$$

Hence, it can be inferred that the mass of Jupiter is about one-thousandth that of the Sun.

Question 8.5:

Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5 ly.

Answer

Mass of our galaxy Milky Way, $M = 2.5 \times 10^{11}$ solar mass

Solar mass = Mass of Sun = 2.0×10^{36} kg

Mass of our galaxy, $M = 2.5 \times 10^{11} \times 2 \times 10^{36} = 5 \times 10^{41}$ kg

Diameter of Milky Way, $d = 10^5$ ly

Radius of Milky Way, $r = 5 \times 10^4$ ly

1 ly = 9.46×10^{15} m

$\therefore r = 5 \times 10^4 \times 9.46 \times 10^{15}$

$= 4.73 \times 10^{20}$ m

Since a star revolves around the galactic centre of the Milky Way, its time period is given by the relation:

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

$$= \left(\frac{4 \times (3.14)^2 \times (4.73)^3 \times 10^{60}}{6.67 \times 10^{-11} \times 5 \times 10^{41}}\right)^{\frac{1}{2}} = \left(\frac{39.48 \times 105.82 \times 10^{30}}{33.35}\right)^{\frac{1}{2}}$$

$$= \left(125.27 \times 10^{30}\right)^{\frac{1}{2}} = 1.12 \times 10^{16} \text{ s}$$

$$1 \text{ year} = 365 \times 324 \times 60 \times 60 \text{ s}$$

$$1 \text{ s} = \frac{1}{365 \times 24 \times 60 \times 60} \text{ years}$$

$$\therefore 1.12 \times 10^{16} \text{ s} = \frac{1.12 \times 10^{16}}{365 \times 24 \times 60 \times 60}$$

$$= 3.55 \times 10^{8} \text{ years}$$

Question 8.6:

Choose the correct alternative:

If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Answer

Answer:

Kinetic energy

Less

Total mechanical energy of a satellite is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the gravitational potential energy of the satellite is zero. As the Earth-satellite system is a bound system, the total energy of the satellite is negative.

Thus, the total energy of an orbiting satellite at infinity is equal to the negative of its kinetic energy.

An orbiting satellite acquires a certain amount of energy that enables it to revolve around the Earth. This energy is provided by its orbit. It requires relatively lesser energy to move out of the influence of the Earth's gravitational field than a stationary object on the Earth's surface that initially contains no energy.

Question 8.7:

Does the escape speed of a body from the earth depend on

the mass of the body,

the location from where it is projected,

the direction of projection,

the height of the location from where the body is launched?

Answer

No

No

No

Yes

Escape velocity of a body from the Earth is given by the relation:

$$v_{esc} = \sqrt{2gR} \qquad \text{... (i)}$$

g = Acceleration due to gravity

R = Radius of the Earth

It is clear from equation (i) that escape velocity v_{esc} is independent of the mass of the body and the direction of its projection. However, it depends on gravitational potential at the point from where the body is launched. Since this potential marginally depends on the height of the point, escape velocity also marginally depends on these factors.

Question 8.8:

A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Answer

No

No

Yes

No

No

Yes

Angular momentum and total energy at all points of the orbit of a comet moving in a highly elliptical orbit around the Sun are constant. Its linear speed, angular speed, kinetic, and potential energy varies from point to point in the orbit.

Question 8.9:

Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem?

Answer

Answer: (b), (c), and (d)

Legs hold the entire mass of a body in standing position due to gravitational pull. In space, an astronaut feels weightlessness because of the absence of gravity. Therefore, swollen feet of an astronaut do not affect him/her in space.

A swollen face is caused generally because of apparent weightlessness in space. Sense organs such as eyes, ears nose, and mouth constitute a person's face. This symptom can affect an astronaut in space.

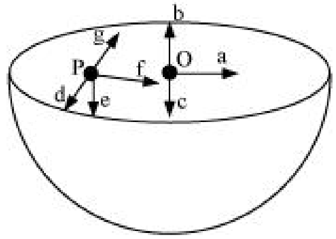
Headaches are caused because of mental strain. It can affect the working of an astronaut in space.

Space has different orientations. Therefore, orientational problem can affect an astronaut in space.

Question 8.10:

Choose the correct answer from among the given ones:

The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12) (i) a, (ii) b, (iii) c, (iv) O.

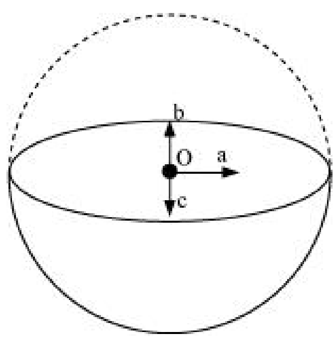


Answer

Answer: (iii)

Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle located at centre O will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at centre O of the given hemispherical shell has the direction as indicated by arrow **c**.

Question 8.11:

Choose the correct answer from among the given ones:

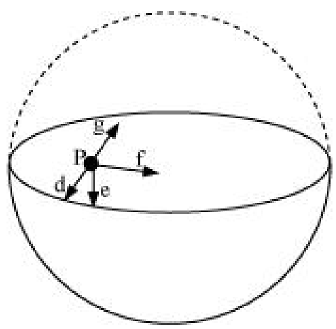
For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Answer

Answer: (ii)

Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient $\left(\frac{dV}{dr}\right)$ is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle at an arbitrary point P will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at an arbitrary point P of the hemispherical shell has the direction as indicated by arrow **e**.

Question 8.12:

A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

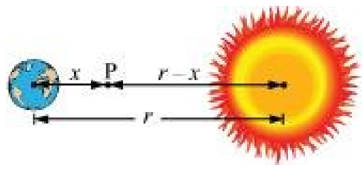
Answer

Mass of the Sun, $M_s = 2 \times 10^{30}$ kg

Mass of the Earth, $M_e = 6 \times 10^{24}$ kg

Orbital radius, $r = 1.5 \times 10^{11}$ m

Mass of the rocket = m



Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton's law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = Gm\frac{M_e}{x^2}$$

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\frac{r-x}{x} = \left(\frac{2\times10^{30}}{60\times10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$1.5\times10^{11} - x = 577.35x$$

$$578.35x = 1.5\times10^{11}$$

$$x = \frac{1.5\times10^{11}}{578.35} = 2.59\times10^{8}\ \text{m}$$

Question 8.13:

How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^{8} km.

Answer

Orbital radius of the Earth around the Sun, $r = 1.5 \times 10^{11}$ m

Time taken by the Earth to complete one revolution around the Sun,

T = 1 year = 365.25 days

= 365.25 × 24 × 60 × 60 s

Universal gravitational constant, G = 6.67×10^{-11} Nm^2 kg^{-2}

Thus, mass of the Sun can be calculated using the relation,

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$= \frac{133.24 \times 10}{6.64 \times 10^4} = 2.0 \times 10^{30} \text{ kg}$$

Hence, the mass of the Sun is 2×10^{30} kg.

Question 8.14:

A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun?

Answer

Distance of the Earth from the Sun, $r_e = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

Time period of the Earth = T_e

Time period of Saturn, T_s = 29. 5 T_e

Distance of Saturn from the Sun = r_s

From Kepler's third law of planetary motion, we have

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

For Saturn and Sun, we can write

$$\frac{r_s^3}{r_e^3} = \frac{T_s^2}{T_e^2}$$

$$r_s = r_e\left(\frac{T_s}{T_e}\right)^{\frac{2}{3}}$$

$$= 1.5\times10^{11}\left(\frac{29.5 T_e}{T_e}\right)^{\frac{2}{3}}$$

$$= 1.5\times10^{11}(29.5)^{\frac{2}{3}}$$

$$= 1.5\times10^{11}\times9.55$$

$$= 14.32\times10^{11}\,\text{m}$$

Hence, the distance between Saturn and the Sun is 1.43×10^{12} m..

Question 8.15:

A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Answer

Weight of the body, $W = 63$ N

Acceleration due to gravity at height h from the Earth's surface is given by the relation:

$$g' = \frac{g}{\left(\frac{1+h}{R_e}\right)^2}$$

Where,

g = Acceleration due to gravity on the Earth's surface

*R*e = Radius of the Earth

$$For\ h = \frac{R_e}{2}$$

$$g' = \frac{g}{\left(1 + \frac{R_e}{2 \times R_e}\right)^2} = \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{4}{9} g$$

Weight of a body of mass *m* at height *h* is given as:

$$\begin{aligned} W' &= mg' \\ &= m \times \frac{4}{9} g = \frac{4}{9} \times mg \\ &= \frac{4}{9} W \\ &= \frac{4}{9} \times 63 = 28 \text{ N} \end{aligned}$$

Question 8.16:

Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

Answer

Weight of a body of mass *m* at the Earth's surface, $W = mg = 250$ N

Body of mass *m* is located at depth, $d = \frac{1}{2} R_e$

Where,

R_e = Radius of the Earth

Acceleration due to gravity at depth g (d) is given by the relation:

$$g' = \left(1 - \frac{d}{R_e}\right) g$$
$$= \left(1 - \frac{R_e}{2 \times R_e}\right) g = \frac{1}{2} g$$

Weight of the body at depth d,

$$W' = mg'$$
$$= m \times \frac{1}{2} g = \frac{1}{2} mg = \frac{1}{2} W$$
$$= \frac{1}{2} \times 250 = 125 \text{ N}$$

Question 8.17:

A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 × 10^{24} kg; mean radius of the earth = 6.4 × 10^{6} m; G= 6.67 × 10^{-11} N m^{2} kg$^{-2.}$

<u>Answer</u>

Answer: 8 × 10^{6} m from the centre of the Earth

Velocity of the rocket, v = 5 km/s = 5 × 10^{3} m/s

Mass of the Earth, $M_e = 6.0 \times 10^{24}$ kg

Radius of the Earth, $R_e = 6.4 \times 10^{6}$ m

Height reached by rocket mass, $m = h$

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e}\right)$$

At highest point *h*,

$v = 0$

And, Potential energy $= -\frac{GM_e m}{R_e + h}$

Total energy of the rocket $= 0 + \left(-\frac{GM_e m}{R_e + h}\right) = -\frac{GM_e m}{R_e + h}$

From the law of conservation of energy, we have

Total energy of the rocket at the Earth's surface = Total energy at height *h*

$$\frac{1}{2}mv^2 + \left(-\frac{GM_e m}{R_e}\right) = -\frac{GM_e m}{R_e + h}$$

$$\frac{1}{2}v^2 = GM_e\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$$

$$= GM_e\left(\frac{R_e + h - R_e}{R_e(R_e + h)}\right)$$

$$\frac{1}{2}v^2 = \frac{GM_e h}{R_e(R_e + h)} \times \frac{R_e}{R_e}$$

$$\frac{1}{2} \times v^2 = \frac{gR_e h}{R_e + h}$$

Where $g = \frac{GM}{R_e^2} = 9.8\ \text{m/s}^2$ (Acceleration due to gravity on the Earth's surface)

$$\therefore v^2(R_e + h) = 2gR_e h$$

$$v^2 R_e = h(2gR_e - v^2)$$

$$h = \frac{R_e v^2}{2gR_e - v^2}$$

$$= \frac{6.4 \times 10^6 \times (5 \times 10^3)^2}{2 \times 9.8 \times 6.4 \times 10^6 - (5 \times 10^3)^2}$$

$$h = \frac{6.4 \times 25 \times 10^{12}}{100.44 \times 10^6} = 1.6 \times 10^6\ \text{m}$$

Height achieved by the rocket with respect to the centre of the Earth

$= R_e + h$
$= 6.4\times10^6 + 1.6\times10^6$
$= 8.0\times10^6\ \text{m}$

Question 8.18:

The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Answer

Escape velocity of a projectile from the Earth, v_{esc} = 11.2 km/s

Projection velocity of the projectile, $v_p = 3v_{esc}$

Mass of the projectile = m

Velocity of the projectile far away from the Earth = v_f

Total energy of the projectile on the Earth $= \frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth = $\frac{1}{2}mv_f^2$

From the law of conservation of energy, we have

$$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{v_p^2 - v_{esc}^2}$$

$$= \sqrt{(3v_{esc})^2 - (v_{esc})^2}$$

$$= \sqrt{8}\ v_{esc}$$

$$= \sqrt{8}\times11.2 = 31.68\ \text{km/s}$$

Question 8.19:

A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^{6} m; G = 6.67×10^{-11} N m^2 kg^{-2}.

Answer

Mass of the Earth, $M = 6.0 \times 10^{24}$ kg

Mass of the satellite, $m = 200$ kg

Radius of the Earth, $R_e = 6.4 \times 10^6$ m

Universal gravitational constant, G = 6.67×10^{-11} Nm^{2}kg^{-2}

Height of the satellite, $h = 400$ km $= 4 \times 10^5$ m $= 0.4 \times 10^6$ m

Total energy of the satellite at height h $= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$

Orbital velocity of the satellite, $v = \sqrt{\frac{GM_e}{R_e + h}}$

Total energy of height, h $= \frac{1}{2}m\left(\frac{GM_e}{R_e + h}\right) - \frac{GM_e m}{R_e + h} = -\frac{1}{2}\left(\frac{GM_e m}{R_e + h}\right)$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

Energy required to send the satellite out of its orbit = – (Bound energy)

$$= \frac{1}{2} \frac{GM_e m}{(R_e + h)}$$

$$= \frac{1}{2} \times \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{\left(6.4 \times 10^6 + 0.4 \times 10^6\right)}$$

$$= \frac{1}{2} \times \frac{6.67 \times 6 \times 2 \times 10}{6.8 \times 10^6} = 5.9 \times 10^9 \text{ J}$$

Question 8.20:

Two stars each of one solar mass (= 2× 10^{30} kg) are approaching each other for a head on collision. When they are a distance 109 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 104 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Answer

Mass of each star, $M = 2 \times 10^{30}$ kg

Radius of each star, $R = 10^4$ km = 10^7 m

Distance between the stars, $r = 10^9$ km = 10^{12}m

For negligible speeds, $v = 0$ total energy of two stars separated at distance r

$$= \frac{-GMM}{r} + \frac{1}{2} mv^2$$

$$= \frac{-GMM}{r} + 0 \qquad \text{... (i)}$$

Now, consider the case when the stars are about to collide:

Velocity of the stars = v

Distance between the centers of the stars = $2R$

Total kinetic energy of both stars $= \frac{1}{2} Mv^2 + \frac{1}{2} Mv^2 = Mv^2$

Total potential energy of both stars $= \dfrac{-GMM}{2R}$

Total energy of the two stars = $Mv^2 - \dfrac{GMM}{2R}$ ... (ii)

Using the law of conservation of energy, we can write:

$$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{r}$$
$$v^2 = \frac{-GM}{r} + \frac{GM}{2R} = GM\left(-\frac{1}{r} + \frac{1}{2R}\right)$$
$$= 6.67\times10^{-11}\times2\times10^{30}\left[-\frac{1}{10^{12}} + \frac{1}{2\times10^{7}}\right]$$
$$= 13.34\times10^{19}\left[-10^{-12} + 5\times10^{-8}\right]$$

$$\sim 13.34\times10^{19}\times5\times10^{-8}$$
$$\sim 6.67\times10^{12}$$
$$v = \sqrt{6.67\times10^{12}} = 2.58\times10^{6}\,\text{m/s}$$

Question 8.21:

Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centers of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Answer

Answer:

0;

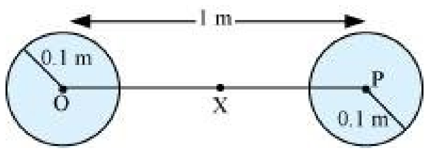
-2.7×10^{-8} J /kg;

Yes;

Unstable

Explanation:

The situation is represented in the given figure:



Mass of each sphere, M = 100 kg

Separation between the spheres, r = 1m

X is the mid point between the spheres. Gravitational force at point X will be zero. This is because gravitational force exerted by each sphere will act in opposite directions.

Gravitational potential at point *X:*

$$= \frac{-GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)} = -4\frac{GM}{r}$$

$$= \frac{4 \times 6.67 \times 10^{-11} \times 100}{1}$$

$$= -2.67 \times 10^{-8} \text{ J/kg}$$

Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.

Question 8.22:

As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km.

Answer

Mass of the Earth, $M = 6.0 \times 10^{24}$ kg

Radius of the Earth, $R = 6400$ km $= 6.4 \times 10^{6}$ m

Height of a geostationary satellite from the surface of the Earth,

$h = 36000$ km $= 3.6 \times 10^{7}$ m

Gravitational potential energy due to Earth's gravity at height *h,*

$$= \frac{-GM}{(R+h)}$$

$$= -\frac{6.67\times10^{-11}\times6.0\times10^{24}}{3.6\times10^{7}+0.64\times10^{7}}$$
$$= -\frac{6.67\times6}{4.24}\times10^{13-7}$$
$$= -9.4\times10^{6} \text{ J/kg}$$

Question 8.23:

A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun = 2×10^{30} kg).

Answer

Answer: Yes

A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $f_g = \frac{GMm}{R^2}$

Where,

M = Mass of the star $= 2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$ kg

m = Mass of the body

R = Radius of the star = 12 km = 1.2 $\times 10^4$ m

$$\therefore f_g = \frac{6.67\times10^{-11}\times5\times10^{30}\times m}{\left(1.2\times10^{4}\right)^{2}} = 2.31\times10^{11} m \text{ N}$$

Centrifugal force, $f_c = mr\omega^2$
ω = Angular speed = $2\pi\nu$

ν = Angular frequency = 1.2 rev s^{-1}

$f_c = mR\,(2\pi\nu)^2$

$= m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2 = 1.7 \times 10^5 m$ N

Since $f_g > f_c$, the body will remain stuck to the surface of the star.

Question 8.24:

A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the Sun = 2×10^{30} kg; mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28×10^{8}kg; G= 6.67×10^{-11} m^2kg^{-2}.

Answer

Mass of the spaceship, m_s = 1000 kg

Mass of the Sun, $M = 2 \times 10^{30}$ kg

Mass of Mars, $m_m = 6.4 \times 10^{23}$ kg

Orbital radius of Mars, $R = 2.28 \times 10^8$ kg $= 2.28 \times 10^{11}$m

Radius of Mars, r = 3395 km = 3.395×10^6 m

Universal gravitational constant, G = 6.67×10^{-11} m^2kg^{-2}

Potential energy of the spaceship due to the gravitational attraction of the Sun $= \frac{-GMm_s}{R}$

Potential energy of the spaceship due to the gravitational attraction of Mars $= \frac{-GM_m m_s}{r}$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship $= \frac{-GMm_s}{R} - \frac{-GM_s m_m}{r}$

$$= -Gm_s\left(\frac{M}{R} + \frac{m_m}{r}\right)$$

The negative sign indicates that the system is in bound state.

Energy required for launching the spaceship out of the solar system

= – (Total energy of the spaceship)

$$= Gm_s\left(\frac{M}{R} + \frac{m_m}{r}\right)$$
$$= 6.67\times10^{-11}\times10^3\times\left(\frac{2\times10^{30}}{2.28\times10^{11}} + \frac{6.4\times10^{23}}{3.395\times10^6}\right)$$
$$= 6.67\times10^{-8}\left(87.72\times10^{17} + 1.88\times10^{17}\right)$$
$$= 6.67\times10^{-8}\times89.50\times10^{17}$$
$$= 596.97\times10^9$$
$$= 6\times10^{11}\,\text{J}$$

Question 8.25:

A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s–1. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4× 1023 kg; radius of mars = 3395 km; G = 6.67× 10^{-11} N m^2 kg^{-2}.

Answer

Initial velocity of the rocket, $v = 2$ km/s $= 2 \times 10^3$ m/s

Mass of Mars, $M = 6.4 \times 10^{23}$ kg

Radius of Mars, $R = 3395$ km $= 3.395 \times 10^6$ m

Universal gravitational constant, G = 6.67×10^{-11} N m^2 kg^{-2}

Mass of the rocket = m

Initial kinetic energy of the rocket = $\frac{1}{2}mv^2$

Initial potential energy of the rocket $= \frac{-GMm}{R}$

Total initial energy $= \frac{1}{2}mv^2 - \frac{GMm}{R}$

If 20 % of initial kinetic energy is lost due to Martian atmospheric resistance, then only 80 % of its kinetic energy helps in reaching a height.

Total initial energy available $= \frac{80}{100} \times \frac{1}{2}mv^2 - \frac{GMm}{R} = 0.4mv^2 - \frac{GMm}{R}$

Maximum height reached by the rocket = h

At this height, the velocity and hence, the kinetic energy of the rocket will become zero.

Total energy of the rocket at height h $= -\frac{GMm}{(R+h)}$

Applying the law of conservation of energy for the rocket, we can write:

$$0.4mv^2 - \frac{GMm}{R} = \frac{-GMm}{(R+h)}$$

$$0.4v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$= GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= GM\left(\frac{R+h-R}{R(R+h)}\right)$$

$$= \frac{GMh}{R(R+h)}$$

$$\frac{R+h}{h} = \frac{GM}{0.4v^2R}$$

$$\frac{R}{h} + 1 = \frac{GM}{0.4v^2R}$$

$$\frac{R}{h} = \frac{GM}{0.4v^2R} - 1$$

$$h = \frac{R}{\frac{GM}{0.4v^2R} - 1}$$

$$= \frac{0.4R^2v^2}{GM - 0.4v^2R}$$

$$= \frac{0.4\times\left(3.395\times10^6\right)^2\times\left(2\times10^3\right)^2}{6.67\times10^{-11}\times6.4\times10^{23} - 0.4\times\left(2\times10^3\right)^2\times\left(3.395\times10^6\right)}$$

$$= \frac{18.442\times10^{18}}{42.688\times10^{12} - 5.432\times10^{12}} = \frac{18.442}{37.256}\times10^6$$

$$= 495\times10^3 \text{ m} = 495 \text{ km}$$

UNIT – VI

GRAVITATION

- **Newton's law of gravitation.** It states that the gravitational force of attraction acting between two bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them, i.e., $F = G\frac{m_1 m_2}{r^2}$; where G is the universal gravitational constant.

  The value of $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

- **Gravity :** It is the force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth.

- **Acceleration due to gravity (g).** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone. It is a vector quantity.

$$g = \frac{GM}{R^2}$$

  where M and R are the mass and radius of the earth.

- **Variation of acceleration due to gravity.**

  (i) Effect of altitude, $g' = \frac{g R^2}{(R+h)^2}$ and $g' = g\left(1 - \frac{2h}{R}\right)$

  The first relation is valid when h is comparable with R and the second relation is valid when h < < R. The value of g decreases with increase in h.

  (ii) **Effect of depth,** $g' = g\left(1 - \frac{d}{R}\right)$

  The acceleration due to gravity decreases with increase in depth d and becomes zero at the centre of earth.

- **Gravitational field.** It is the space around a material body in which its gravitational pull can be experienced by other bodies.

  The intensity of gravitational field at a point at a distance r from the centre of the body of mass M is given by $I = GM/r^2 = g$ (acceleration due to gravity).

- **Gravitational potential.** The gravitational potential at a point in a gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

  Gravitational potential at a point, $V = \frac{\text{work done}(W)}{\text{test mass}(m_0)} = \frac{GM}{r}$

- Gravitational potential energy U = gravitational potential × mass of body

  $$= -\frac{GM}{r} \times m$$

  Gravitational intensity (I) is related to gravitational potential (V) at a point by the relation, $-\frac{dv}{dr}$

- **Satellite.** A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.

  (i) Orbital speed of a satellite when it is revolving around earth at height h is given by

  $$v_0 = R\sqrt{\frac{g}{R+h}}$$

  When the satellite is orbiting close to the surface of earth, i.e., h < < R, then

  $$v_0 = R\sqrt{\frac{g}{R}} = \sqrt{g\,R}$$

  (ii) Time period of satellite *(T)*. It is the time taken by the satellite to complete one revolution around the earth.

  $$T = \frac{2\pi(R+h)}{v_0} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

(iii) Height of satellite above the earth's surface :

$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

(iv) Total energy of satellite, E = P.E. + K.E.

$$E = -\frac{GM\,m}{(R+h)} + \frac{1}{2}mv_0^2 = -\frac{GM\,m}{(R+h)} + \frac{1}{2}m\left(\frac{GM}{R+h}\right) = -\frac{GM\,m}{2(R+h)}$$

If the satellite is orbiting close to earth, then r = R. Now total energy of satellite.

$$E = -\frac{GM\,m}{2R}$$

(v) **Binding energy of satellite.** $= -E = \dfrac{GM\,m}{2r}$

- **Escape speed.** The escape speed on earth is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth so that it just crosses the gravitational field of earth. Escape velocity ν_e is given by,

$$\nu_e = \sqrt{\frac{2GM}{R}} = \sqrt{2\,g\,R}$$

For earth, the value of escape speed is 11.2 kms^{-1}.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Q 1. The mass of moon is nearly 10% of the mass of the earth. What will be the gravitational force of the earth on the moon, in comparison to the gravitational force of the moon on the earth?

Q 2. Why does one feel giddy while moving on a merry round?

Q 3. Name two factors which determine whether a planet would have atmosphere or not.

Q 4. The force of gravity due to earth on a body is proportional to its mass, then why does a heavy body not fall faster than a lighter body?

Q 5. The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside is zero, so can a body be shielded from gravitational influence?

Q 6. The gravitational force between two bodies in 1 N if the distance between them is doubled, what will be the force between them?

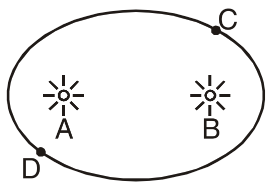
Q 7. A body of mass 5 kg is taken to the centre of the earth. What will be its (i) mass (ii) weight there.

Q 8. Why is gravitational potential energy negative?

Q 9. A satellite revolves close to the surface of a planet. How is its orbital velocity related with escape velocity of that planet.

Q 10. Does the escape velocity of a body from the earth depend on (i) mass of the body (ii) direction of projection

Q 11. Identify the position of sun in the following diagram if the linear speed of the planet is greater at C than at D.



Q 12. A satellite does not require any fuel to orbit the earth. Why?

Q 13. A satellite of small mass burns during its desent and not during ascent. Why?

Q 14. Is it possible to place an artificial satellite in an orbit so that it is always visible over New Delhi?

Q 15. If the density of a planet is doubled without any change in its radius, how does 'g' change on the planet.

Q 16. Why is the weight of a body at the poles more than the weight at the equator? Explain.

Q 17. Why an astronaut in an orbiting space craft is not in zero gravity although weight less?

Q 18. Write one important use of (i) geostationary satellite (ii) polar satellite.

Q 19. A binary star system consists of two stars A and B which have time periods T_A and T_B, radius R_A and R_B and masses m_A and m_B which of the three quantities are same for the stars. Justify.

Q 20. The time period of the satellite of the earth is 5 hr. If the separation between earth and satellite is increased to 4 times the previous value, then what will be the new time period of satellite.

Q 21. Why does the earth impart the same acceleration to antibodies?

Q 22. If suddenly the gravitational force of attraction between earth and satellite become zero, what would happen to the satellite?

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Q 1. If the radius of the earth were to decrease by 1%, keeping its mass same, how will the acceleration due to gravity change?

Q 2. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

Q 3. A satellite is moving round the earth with velocity v_0 what should be the minimum percentage increase in its velocity so that the satellite escapes.

Q 4. The radii of two planets are R and 2R respectively and their densities ρ and $\rho/2$ respectively. What is the ratio of acceleration due to gravity at their surfaces?

Q 5. If earth has a mass 9 times and radius 4 times than that of a planet 'P'. Calculate the escape velocity at the planet 'P' if its value on earth is 11.2 kms^{-1}

Q 6. At what height from the surface of the earth will the value of 'g' be reduced by 36% of its value at the surface of earth.

Q 7. At what depth is the value of 'g' same as at a height of 40 km from the surface of earth.

Q 8. The mean orbital radius of the earth around the sun is 1.5×10^8 km. Calculate mass of the sun if $G = 6.67 \times 10^{-11}$ N m^2/kg^{-2}?

Q 9. Draw graphs showing the variation of acceleration due to gravity with (i) height above earth is surface (ii) depth below the earth's surface.

Q 10. A rocket is fired from the earth towards the sun. At what point on its path is the gravitational force on the rocket zero? Mass of sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. Orbital radius = 1.5×10^{11} m.

Q 11. A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^{8} km away from the sun?

Q 12. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

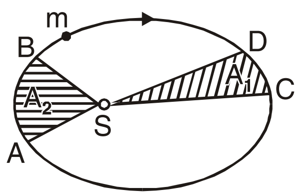
Q 13. Why the space rockets are generally launched west to East?

Q 14. Explain why a tennis ball bounces higher on hills than in plane?

Q 15. The gravitational force on the earth due to the sun is greater than moon. However tidal effect due to the moon's pull is greater than the tidal effect due to sun. Why?

Q 16. The mass of moon is $\frac{M}{81}$ (where M is mass of earth). Find the distance of the point where the gravitational field due to earth and moon cancel each other. Given distance of moon from earth is 60 R, where R is radius of earth.

Q 17. The figure shows elliptical orbit of a planet m about the sun S. The shaded area of SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from D to C and t_2 is time to move from A to B, what is the relation between t_1 and t_2?



Q 18. Calculate the energy required to move a body of mass m from an orbit of radius 2R to 3R.

Q 19. A man can jump 1.5 m high on earth. Calculate the height he may be able to jump on a planet whose density is one quarter that of the earth and whose radius is one third of the earth.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

Q 1. Define gravitational potential at a point in the gravitational field. Obtain a relation for it. What is the position at which it is (i) maximum (ii) minimum.

Q 2. Find the potential energy of a system of four particles, each of mass m, placed at the vertices of a square of side. Also obtain the potential at the centre of the square.

Q 3. Three mass points each of mass m are placed at the vertices of an equilateral triangle of side l. What is the gravitational field and potential at the centroid of the triangle due to the three masses.

Q 4. Briefly explain the principle of launching an artificial satellite. Explain the use of multistage rockets in launching a satellite.

Q 5. In a two stage launch of a satellite, the first stage brings the satellite to a height of 150 km and the 2^{nd} stage gives it the necessary critical speed to put it in a circular orbit. Which stage requires more expenditure of fuel? Given mass of earth = 6.0×10^{24} kg, radius of earth = 6400 km

Q 6. The escape velocity of a projectile on earth's surface is 11.2 kms^{-1} A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Q 7. A satellite orbits the earth at a height 'R' from the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence?

Q 8. Define gravitational potential. Give its SI units.

Q 9. What do you mean by gravitational potential energy of a body? Obtain an expression for it for a body of mass m lying at distance r from the centre of the earth.

Q 10. State and explain Kepler's laws of planetary motion. Name the physical quantities which remain constant during the planetary motion.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

Q 1. What is acceleration due to gravity?

Obtain relations to show how the value of 'g' changes with (i) attitude

(ii) depth

Q 2. Define escape velocity obtain an expression for escape velocity of a body from the surface of earth? Does the escape velocity depend on (i) location from where it is projected (ii) the height of the location from where the body is launched.

Q 3. State Kepler's three laws of planetary motion. Prove the second and third law.

Q 4. Derive expression for the orbital velocity of a satellite and its time period.

What is a geostatinary satellite. Obtain the expression for the height of the geostationary satellite.

Q 5. State and derive Kepler's law of periods (or harmonic law) for circular orbits.

Q 6. A black hole is a body from whose surface nothing may ever escape. What is the condition for a uniform spherical mass M to be a black hole? What should be the radius of such a black hole if its mass is the same as that of the earth?

NUMERICALS

Q 1. The mass of planet Jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30}kg. The mean distance of Jupiter from the Sun is 7.8×10^{11}m. Calculate gravitational force which sun exerts on Jupiter, and the speed of Jupiter.

Q 2. A mass 'M' is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum.

Q 3. If the radius of earth shrinks by 2%, mass remaining constant. How would the value of acceleration due to gravity change ?

Q 4. Find the value of g at a height of 400 km above the surface of the earth. Given radius of the earth, R = 6400 km and value of g at the surface of the earth = 9.8 ms^{-2}. [**Ans.** 8.575 ms^{-2}]

Q 5. How far away from the surface of earth does the acceleration due to gravity become 4% of its value on the surface of earth? Radius of earth = 6400 km. [**Ans.** 25,600 km]

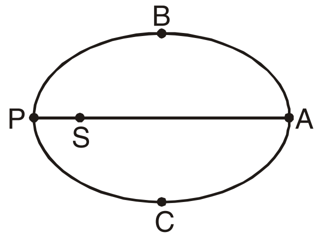
Q 6. The gravitational field intensity at a point 10,000 km from the centre of the earth is 4.8 N kg^{-1}. Calculate gravitational potential at that point.

Q 7. A geostationary satellite orbits the earth at a height of nearly 36000 km. What is the potential due to earth's gravity at the site of this satellite (take the potential energy at ∞ to be zero). Mass of earth is 6×10^{24} kg, radius of earth is 6400 km.

Q 8. Jupiter has a mass 318 times that of the earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface, given that the escape velocity from the earth's surface is 11.2 km s^{-1}.

Q 9. The distance of Neptune and Saturn from the sun is nearly 10^{13}m and 10^{12}m respectively. Assuming that they move in circular orbits, then what will be the ratio of their periods.

Q 10. Let the speed of the planet at perihelion P in fig be v_p and Sun planet distance SP be r_p Relate (r_p, v_p) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse BAC and CPB?



ANSWER FOR VERY SHORT QUESTIONS (1 MARK)

1. Both forces will be equal in magnitude as gravitational force is a mutual force between the two bodies.

2. When moving in a merry go round, our weight appears to decrease when we move down and increases when we move up, this change in weight makes us feel giddy.

3. (i) Value of acceleration due to gravity (ii) surface temperature of planet.

4. $\because F = \frac{GMm}{R^2}$ F α m but $g = \frac{Gm}{R^2}$ and does not depend on 'm' hence

they bodies fall with same 'g'.

5. No, the gravitational force is independent of intervening medium.

6. F = 1 $F' = \frac{F}{4}$

7. Mass does not change.

8. Because it arises due to attractive force of gravitation.

9. $\nu_e = \sqrt{2}\ \nu_o \quad \because \ \nu_e = \sqrt{\frac{2\,GM}{R}} \quad \nu_o = \sqrt{\frac{GM}{R}}$ when $r = R$

10. No, $\nu_e = \sqrt{\frac{2\,GM}{R}}$

11. Sun should be at B as speed of planet is greater when it is closer to sun.

12. The gravitational force between satellite and earth provides the necessary centripetal force for the satellite to orbit the earth.

13. The speed of satellite during descent is much larger than during ascent, and so heat produced is large.

14. No, A satellite will be always visible only if it revolves in the equatorial plane, but New Delhi does not lie in the region of equitorial plane.

15. 'g' gets doubled as $g \propto \rho$ (density)

16. As $g = GM/R^2$ and the value of R at the poles is less than that the equator, so g at poles is greater that g at the equator. Now, $g_p > g_e$, hence $mg_p > g_e$ i.e., the weight of a body at the poles is more than the weight at the equator.

17. The astronaut is in the gravitational field of the earth and experiences gravity. However, the gravity is used in providing necessary centripetal force, so is in a state of free fall towards the earth.

18. Geostationary satellite are used for tele communication and polar satellite for remote rensing.

19. Angular velocity of binary stars are same is $w_A = w_B$,

$$\frac{2\pi}{T_A} = \frac{2\pi}{T_B} \quad \Rightarrow \quad T_A = T_B$$

20. $\frac{T_2^2}{T_1^2} = \left(\frac{R_2}{R_1}\right)^3 \Rightarrow T_2^2 = 64 \times 25 \Rightarrow T_2 = 40\,\text{hr}$

21. The force of gravitation exerted by the earth on a body of mass m is $F = G\frac{Mm}{R^2} = mg$.

    Acceleration imparted to the body, $g = \frac{GM}{R^2}$

    Clearly, g does not depend on m. Hence the earth imparts same acceleration to all bodies.

22. The satellite will move tangentially to the original orbit with a velocity with which it was revolving.

SHORT ANSWERS (2 MARKS)

1. $g = \frac{GM}{R^2}$ If R decreases by 1% it becomes $\frac{99}{100}R$

    $$\therefore g' = \frac{GM}{(.99\,R)^2} = 1.02\frac{GM}{R^2} = (1+0.02)\frac{GM}{R^2}$$

    $\therefore$ g' increases by 0.02 $\frac{GM}{R^2}$, therefore increases by 2%.

2. (b), (c) and (d) are affected in space.

3. The maximum orbital velocity of a satellite orbiting near its surface is $\nu_o = \sqrt{gR} = \frac{\nu_e}{\sqrt{2}}$

    For the satellite to escape gravitational pull the velocity must become v_e

    But $\nu_e = \sqrt{2}\nu_o = 1.414\ \nu_o = (1+0.414)\nu_o$

    This means that it has to increases 0.414 in 1 or 41.4%.

$\therefore$ The minimum increase required, as the velocity of satellite is maximum when it is near the earth.

4. Here

$$g = \frac{GM}{R^2} = \frac{GM}{R^2} \cdot \frac{4}{3} \pi R^3 \rho$$

or $\quad g \; \alpha \; R\rho$

$$\therefore \quad \frac{g_1}{g_2} = \frac{R\rho}{2R.\frac{\rho}{2}} = 1:1.$$

5. $\nu_e = \sqrt{\frac{2GM}{R_e}} \quad \nu_p = \sqrt{\frac{2\,GM_p}{R_p}} \quad M_p = \frac{M}{9}, R_p = \frac{R_e}{4}$

$$\therefore \; \nu_p = \sqrt{2\,G\frac{M}{9} \times \frac{4}{R_e}} \quad = \quad \frac{2}{3}\sqrt{\frac{2GM}{R_e}} \quad = \quad \frac{2}{3} \times 11.2 = \frac{22.4}{3}$$

$= 7.47$ km/sec

6. g' = 64% of g $= \frac{64}{100} g$

$$g' = g\frac{R^2}{(R+h)^2} = \frac{64}{100} g$$

$$\therefore \; \frac{R}{R+h} = \frac{8}{10}$$

$$h = \frac{R}{4} = 1600\,\text{km}$$

7. $g_d = g_h$

$$g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{2h}{R}\right)$$

d = 2h = 2 × 40 = 80 km

8. $R = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

$T = 365$ days $= 365 \times 24 \times 3600$ s

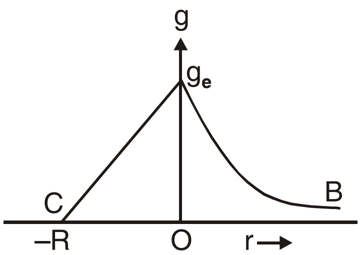
Centripetal force = gravitational force

$$\frac{mv^2}{R} = \frac{GMm}{R^2} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{GMm}{R^2}$$

$$M_s = \frac{4\pi^2 R^3}{G T^2} = \frac{4 \times 9.87 \times \left(1.5 \times 10^{11}\right)^3}{6.64 \times 10^{-11} \times \left(365 \times 24 \times 3600\right)^2}$$

$M_s = 2.01 \times 10^{30}$ kg

9.



$g \propto \frac{1}{r^2}$ for $r > 0$ above surface of earth

$g \propto (R - d)$ for $r < 0$ below surface of earth

g is max for r = 0 on surface.

10. Given $M_s = 2 \times 10^{30}$ kg,

$M_e = 6 \times 10^{24}$ kg, $r = 1.5 \times 10^{11}$ m

Let m be the mass of the rocket. Let at distance x from the earth, the gravitational force on the rocket be zero.

Then at this distance, Gravitational pull of the earth on the rocket = Gravitational pull of the sun on the rocket

i.e., $$\frac{GM_e\ m}{x^2} = \frac{GM_e\ m}{(r - x)^2} \quad \text{or} \quad \frac{(r - x)^2}{x^2} = \frac{M_s}{M_e}$$

or $\quad \dfrac{r - x}{x} = \sqrt{\dfrac{M_s}{M_e}} = \sqrt{\dfrac{2 \times 10^{30}}{6 \times 10^{24}}} = \dfrac{10^3}{\sqrt{3}} = 577.35$

or $\quad r - x = 577.35\,x$

or $\quad 578.35\,x = r = 1.5 \times 10^{11}$ or $x = \dfrac{1.5 \times 10^{11}}{578.35} = 2.59 \times 10^8$ m.

11. According to Kepler's law of periods,

$$\left[\frac{T_S}{T_E}\right]^2 = \left[\frac{r_S}{r_E}\right]^3$$

But $\dfrac{T_S}{T_E} = 29.5$ and $r_E = 1.5 \times 10^8$ km.

$\therefore \quad (29.5)^2 = \left(\dfrac{r_S}{1.5 \times 10^8}\right)^3$

or $\quad r_s = 1.5 \times 10^8 \times (29.5)^{2/3} = 14.32 \times 10^8$ km.

12. Here $mg = 63$ N, $h = R/2$

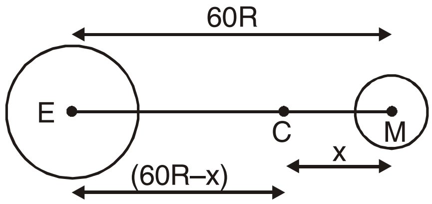
As $\quad \dfrac{g_h}{g} = \left(\dfrac{R}{R+h}\right)^2 = \left(\dfrac{R}{R + \dfrac{R}{2}}\right) = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}.$

$$g_h = \frac{4}{9} g \qquad \therefore \quad mg_h = \frac{4}{9} mg = \frac{4}{9} \times 63 = 28\,\text{N}.$$

13. Since the earth revolves from west to east, so when the rocket is launched from west to east the relative velocity of the rocket increases which helps it to rise without much consumption of fuel.

14. The value of 'g' on hills is less than at the plane, so the weight of tennis ball on the hills is lesser force than at planes that is why the earth attract the ball on hills with lesser force than at planes. Hence the ball bounces higher.

15. The tidal effect depends inversely on the cube of the distance, while gravitational force depends on the square of the distance.

16. 

Gravitational field at C due to earth

= Gravitational field at C due to earth moon

$$\frac{GM}{(60R-x)^2} = \frac{GM/81}{x^2}$$

$$81x^2 = (60R - x)^2$$

$$9x = 60\ R - x$$

$$x = 6\ R$$

17. According to Kepler's IInd law areal velocity for the planet is constant

$$\therefore \qquad \frac{A_1}{t_1} = \frac{A_2}{t_2} \quad A_1 = 2A_2$$

$$\therefore \qquad \frac{2A_2}{t_1} = \frac{A_2}{t_2}$$

$$t_1 = 2t_2$$

18. Gravitational P.E of mass m in orbit of radius R = $U = -\frac{GMm}{R}$

$$\therefore \qquad U_i = -\frac{GMm}{2R}$$

$$U_f = -\frac{GMm}{3R}$$

$$\Delta U = U_f - U_i = GMm\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{GMm}{6R}$$

19. $g = \frac{4}{3}\pi G R \rho$

$$g' = \frac{4}{3}\pi G R' \rho'$$

The gain in P.E at the highest point will be same in both cases. Hence

mg'h' = mgh

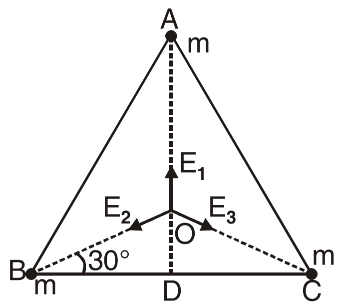
$$h' = \frac{mgh}{mg'} = \frac{m \times \frac{4}{3}\pi G R \rho h}{m \frac{4}{3}\pi G R' \rho'}$$

$$= \frac{R\rho h}{R'\rho'} = \frac{3R' \times 4\rho' \times 1.5}{R' \times \rho'}$$

= 18 m

ANSWERS FOR 3 MARKS QUESTIONS

3.



$$E_1 = \frac{GM}{(OA)^2}$$

$$E_2 = \frac{GM}{(OB)^2}$$

$$E_3 = \frac{GM}{(OC)^2}$$

From Δ ODB Cos 30° $= \frac{BD}{OB} = \frac{\ell/2}{OB}$

$$OB = \frac{\ell/2}{\text{Cos}\,30^\circ} = \frac{\ell}{2}\frac{2}{\sqrt{3}} = \ell/\sqrt{3}$$

Gravitational field at O due to m at A, B and C is say $\overrightarrow{E_1}, \overrightarrow{E_2}$ & $\overrightarrow{E_3}$

$$E = \sqrt{E_2^2 + E_3^2 + 2E_2E_3\text{Cos}120^\circ}$$

$$= \sqrt{\frac{(GM3)^2}{l^2} + \left(\frac{3Gm}{l}\right)^2 + 2\left(\frac{3GM}{l}\right)\left(\frac{3GM}{l}\right)\left(-\frac{1}{2}\right)}$$

$$= \frac{3\,G\,M}{l} \quad \text{along} \quad OD$$

$\bar{E}$ is equal and opposite to $\overrightarrow{E_1}$

$\therefore$ net gravitational field = zero

As gravitational potential is scalar

$V = V_1 + V_2 + V_3$

$$= -\frac{GM}{OA} - \frac{GM}{OB} - \frac{GM}{OC}$$

$$V = -\frac{3GM}{l/\sqrt{3}} = -3\sqrt{3}\frac{Gm}{l}$$

5. Work done on satellite in first stage = W_1 = PE at 150 km – PE at the surface

$$W_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMmh}{R(R+h)}$$

Work done on satellite in 2nd stage = W_2 = energy required to give orbital velocity ν_o

$$= \frac{1}{2} m\nu_0^2 = \frac{1}{2}\left(\frac{GMm}{R+h}\right)$$

$$\frac{W_1}{W_2} = \frac{2h}{R} = \frac{2\times 150}{6400} = \frac{3}{64} < 1$$

$\therefore$ W_2 > W_1 so second stage requires more energy

6. v_e = 11.2 kms^{-1}, velocity of projection = ν = $3\nu_e$ Let m be the mass of projectile and ν_o the velocity after it escapes gravitational pull.

   By law of conservation of energy

$$\frac{1}{2} m\nu_0^2 = \frac{1}{2} m\nu^2 - \frac{1}{2} m\nu_e^2$$

$$\nu_0 = \sqrt{\nu^2 - \nu_e^2} = \sqrt{9\nu_e^2 - \nu_e^2} = \sqrt{8\nu_e^2}$$

$$= 22.4\sqrt{2}$$

$$= 31.68 \text{ km s}^{-1}$$

7. The energy required to pull the satellite from earth influence should be equal to the total energy with which it is revolving around the earth.

   The K.E. of satellite $= \frac{1}{2} m\nu^2 = \frac{1}{2} m \frac{GM}{R+h}$ $\quad \therefore \nu = \sqrt{\frac{GM}{R+h}}$

   The P.E of satellite $= -\frac{GMm}{R+h}$

$$\therefore \text{ T.E.} = \frac{1}{2}\frac{mGM}{(R+h)} - \frac{GMm}{(R+h)} = -\frac{1}{2}\frac{GMm}{(R+h)}$$

$$\therefore \text{ Energy required will be} \left(+\frac{1}{2}\frac{GMm}{(R+h)}\right)$$

ANSWERS FOR NUMERICALS

1. $F = \frac{GMm}{r^2}$

$$= \frac{6.67\times10^{-11}\times1.99\times10^{30}\times1.9\times10^{27}}{\left(7.8\times10^{11}\right)^2}$$

F = 4.1 × 10^{23} N

$$\therefore\ F = \frac{m\nu^2}{r} \quad \Rightarrow \quad \nu = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{GMm}{r^2}\times\frac{r}{m}}$$

$$\nu = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\times1.9\times10^{30}}{7.8\times10^{11}}}$$

ν = 1.3 × 10^4 m s^{-1}

2. Let m_1 = m then m_2 = M – m

   Force between them when they are separated by distance 'r'

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}\left(Mm - m^2\right)$$

   For F to be maximum, differentiate F w.r.t m and equate to zero

$$\frac{dF}{dm} = \frac{G}{r^2}(M-2m) = 0$$

   M = 2m; $m = \frac{M}{2}$

$$\therefore m_1 = m_2 = \frac{M}{2}$$

3. increases by 4%

5. $\frac{4}{100}g = g\left(\frac{R}{R+h}\right)^2$

$$\frac{2}{10} = \frac{R}{R+h}$$

   $\therefore$ h = 4R = 4 × 6400 = 25,600 km.

6. Gravitational intensity = $E = \frac{GM}{R^2}$

Gravitational potential $V = -\frac{GM}{R}$

$\therefore \quad \frac{V}{E} = -R$

or, V = –E × R

or V = –4.8 × 10,000 × 10^3 = -4.8 × 10^7 J kg^{-1}

7. U = Potential at height $h = -\frac{GM}{R+h}$

$$U = -\frac{6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^{6}+36\times10^{6}} = -9.44\times10^{6}\ \text{J/kp}$$

8. Escape velocity from the earth's surface is $\nu_e = \sqrt{\frac{2\ GM}{R}} = 11.2\ \text{kms}^{-1}$

Escape velocity from Jupiter's surface will be

$$\nu'_e = \sqrt{\frac{2\ GM'}{R'}}$$

But M′ = 318 M, R′ = 11.2 R

$$\therefore \qquad \nu'_e = \sqrt{\frac{2\ G\ (318\ M)}{11.2}} = \sqrt{\frac{2GM}{R}\times\frac{318}{11.2}}$$

$$= \nu'_e \times \sqrt{\frac{318}{11.2}} = 11.2 \times \sqrt{\frac{318}{11.2}} = 59.7\ \text{kms}^{-1.}$$

9. By kepler's III^{rd} law

$$\left(\frac{T_n}{T_s}\right)^2 = \left(\frac{R_n}{R_s}\right)^3$$

$$\frac{T_n}{T_5} = \left(\frac{R_n}{R_s}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$$

$$= 10\sqrt{10} = 10\times3.16 = 31.6$$

$\therefore \quad T_n : T_s = 36.6 : 1$

10. The magnitude of angular momentum at P is $L_p = m_p \, r_p \, \nu_p$

Similarly magnitude of angular momentum at A is $L_A = m_A r_A \nu_A$

From conservation of angular momentum

$m_p r_p \nu_p = m_A \nu_A r_A$

$$\frac{\nu_P}{\nu_A} = \frac{r_A}{r_P}$$

$\because \; r_A > r_P, \; \therefore \nu_P > \nu_A$

area bound by SB & SC(SBAC > SBPC)

$\because$ By 2nd law equal areas are swept in equal intervals of time. Time taken to transverse BAC > time taken to traverse CPB