Question 12.1:

Choose the correct alternative from the clues given at the end of the each statement:

(a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

(b) In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force.

(Thomson's model/ Rutherford's model.)

(c) A *classical* atom based on .... is doomed to collapse.

(Thomson's model/ Rutherford's model.)

(d) An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in ....

(Thomson's model/ Rutherford's model.)

(e) The positively charged part of the atom possesses most of the mass in ....

(Rutherford's model/both the models.)

Answer

(a) The sizes of the atoms taken in Thomson's model and Rutherford's model have the same order of magnitude.

(b) In the ground state of Thomson's model, the electrons are in stable equilibrium. However, in Rutherford's model, the electrons always experience a net force.

(c) A *classical* atom based on Rutherford's model is doomed to collapse.

(d) An atom has a nearly continuous mass distribution in Thomson's model, but has a highly non-uniform mass distribution in Rutherford's model.

(e) The positively charged part of the atom possesses most of the mass in both the models.

Question 12.2:

Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

Answer

In the alpha-particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough. This is because the mass of hydrogen (1.67×10^{-27} kg) is less than the mass of incident $\alpha-$particles

(6.64×10^{-27} kg). Thus, the mass of the scattering particle is more than the target nucleus (hydrogen). As a result, the α−particles would not bounce back if solid hydrogen is used in the α-particle scattering experiment.

Question 12.3:

What is the shortest wavelength present in the Paschen series of spectral lines?

Answer

Rydberg's formula is given as:

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^{8} m/s

(n_1 and n_2 are integers)

The shortest wavelength present in the Paschen series of the spectral lines is given for values $n_1 = 3$ and $n_2 = \infty$.

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19}\left[\frac{1}{(3)^2} - \frac{1}{(\infty)^2}\right]$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8} \times 9}{21.76 \times 10^{-19}}$$

$$= 8.189 \times 10^{-7} \text{ m}$$

$$= 818.9 \text{ nm}$$

Question 12.4:

A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Answer

Separation of two energy levels in an atom,

E = 2.3 eV

= $2.3 \times 1.6 \times 10^{-19}$

$= 3.68 \times 10^{-19}$ J

Let ν be the frequency of radiation emitted when the atom transits from the upper level to the lower level.

We have the relation for energy as:

$E = h\nu$

Where,

h = Planck's constant $= 6.62 \times 10^{-34}$ Js

$$\therefore \nu = \frac{E}{h}$$

$$= \frac{3.68\times10^{-19}}{6.62\times10^{-32}} = 5.55\times10^{14}\ \text{Hz}$$

Hence, the frequency of the radiation is 5.6×10^{14} Hz.

Question 12.5:

The ground state energy of hydrogen atom is −13.6 eV. What are the kinetic and potential energies of the electron in this state?

Answer

Ground state energy of hydrogen atom, $E = -13.6$ eV

This is the total energy of a hydrogen atom. Kinetic energy is equal to the negative of the total energy.

Kinetic energy $= -E = -(-13.6) = 13.6$ eV

Potential energy is equal to the negative of two times of kinetic energy.

Potential energy $= -2 \times (13.6) = -27.2$ eV

Question 12.6:

A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of the photon.

Answer

For ground level, $n_1 = 1$

Let E_1 be the energy of this level. It is known that E_1 is related with n_1 as:

$$E_1 = \frac{-13.6}{n_1^2} \text{ eV}$$
$$= \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

The atom is excited to a higher level, $n_2 = 4$.

Let E_2 be the energy of this level.

$$\therefore E_2 = \frac{-13.6}{n_2^2} \text{ eV}$$
$$= \frac{-13.6}{4^2} = -\frac{13.6}{16} \text{ eV}$$

The amount of energy absorbed by the photon is given as:

$E = E_2 - E_1$

$$= \frac{-13.6}{16} - \left(-\frac{13.6}{1}\right)$$
$$= \frac{13.6 \times 15}{16} \text{ eV}$$
$$= \frac{13.6 \times 15}{16} \times 1.6 \times 10^{-19} = 2.04 \times 10^{-18} \text{ J}$$

For a photon of wavelengthλ, the expression of energy is written as:

$$E = \frac{hc}{\lambda}$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^{8} m/s

$$\therefore \lambda = \frac{hc}{E}$$
$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2.04 \times 10^{-18}}$$
$$= 9.7 \times 10^{-8} \text{ m} = 97 \text{ nm}$$

And, frequency of a photon is given by the relation,

$$\nu = \frac{c}{\lambda}$$

$$= \frac{3\times10^{8}}{9.7\times10^{-8}} \approx 3.1 \times 10^{15} \text{ Hz}$$

Hence, the wavelength of the photon is 97 nm while the frequency is 3.1×10^{15} Hz.

Question 12.7:

(a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

Answer

(a) Let v_1 be the orbital speed of the electron in a hydrogen atom in the ground state level, $n_1 = 1$. For charge (e) of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi \in_0 \left(\frac{h}{2\pi}\right)} = \frac{e^2}{2 \in_0 h}$$

Where,

$e = 1.6 \times 10^{-19}$ C

$\in_0$ = Permittivity of free space = 8.85×10^{-12} N^{-1} C^2 m^{-2}

h = Planck's constant = 6.62×10^{-34} Js

$$\therefore v_1 = \frac{\left(1.6\times10^{-19}\right)^2}{2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$= 0.0218 \times 10^{8} = 2.18 \times 10^{6} \ m/s$$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as:

$$v_2 = \frac{e^2}{n_2 2 \in_0 h}$$

$$= \frac{\left(1.6\times10^{-19}\right)^2}{2\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$= 1.09 \times 10^{6} \ m/s$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as:

$$v_3 = \frac{e^2}{n_3 2 \in_0 h}$$
$$= \frac{\left(1.6\times10^{-19}\right)^2}{3\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$
$$= 7.27 \times 10^5 \ m/s$$

Hence, the speed of the electron in a hydrogen atom in $n = 1$, n=2, and n=3 is 2.18×10^6 m/s, 1.09×10^6 m/s, 7.27×10^5 m/s respectively.

(b) Let T_1 be the orbital period of the electron when it is in level $n_1 = 1$.

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{v_1}$$

Where,

r_1 = Radius of the orbit

$$= \frac{n_1^2 h^2 \in_0}{\pi m e^2}$$

h = Planck's constant = 6.62×10^{-34} Js

e = Charge on an electron = 1.6×10^{-19} C

$\in_0$ = Permittivity of free space = 8.85×10^{-12} N^{-1} C^2 m^{-2}

m = Mass of an electron = 9.1×10^{-31} kg

$$\therefore T_1 = \frac{2\pi r_1}{v_1}$$
$$= \frac{2\pi\times(1)^2\times\left(6.62\times10^{-34}\right)^2\times8.85\times10^{-12}}{2.18\times10^6\times\pi\times9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$$
$$= 15.27\times10^{-17} = 1.527\times10^{-16}\ s$$

For level $n_2 = 2$, we can write the period as:

$$T_2 = \frac{2\pi r_2}{v_2}$$

Where,

r_2 = Radius of the electron in n_2 = 2

$$= \frac{(n_2)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\therefore T_2 = \frac{2\pi r_2}{v_2}$$

$$= \frac{2\pi \times (2)^2 \times (6.62\times10^{-34})^2 \times 8.85\times10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1\times10^{-31} \times (1.6\times10^{-19})^2}$$

$$= 1.22\times10^{-15}\ s$$

And, for level n_3 = 3, we can write the period as:

$$T_3 = \frac{2\pi r_3}{v_3}$$

Where,

r_3 = Radius of the electron in n_3 = 3

$$= \frac{(n_3)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\therefore T_3 = \frac{2\pi r_3}{v_3}$$

$$= \frac{2\pi \times (3)^2 \times (6.62\times10^{-34})^2 \times 8.85\times10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1\times10^{-31} \times (1.6\times10^{-19})^2}$$

$$= 4.12 \times 10^{-15}\ s$$

Hence, the orbital period in each of these levels is 1.52×10^{-16} s, 1.22×10^{-15} s, and 4.12×10^{-15} s respectively.

Question 12.8:

The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n =3 orbits?

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Answer

The radius of the innermost orbit of a hydrogen atom, $r_1 = 5.3 \times 10^{-11}$ m.

Let r_2 be the radius of the orbit at $n = 2$. It is related to the radius of the innermost orbit as:

$$r_2 = (n)^2 r_1$$
$$= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}\ m$$

For $n = 3$, we can write the corresponding electron radius as:

$$r_3 = (n)^2 r_1$$
$$= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}\ m$$

Hence, the radii of an electron for $n = 2$ and $n = 3$ orbits are 2.12×10^{-10} m and 4.77×10^{-10} m respectively.

Question 12.9:

A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Answer

It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is −13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes −13.6 + 12.5 eV i.e., −1.1 eV.

Orbital energy is related to orbit level (n) as:

$$E = \frac{-13.6}{(n)^2}\ \text{eV}$$

For $n = 3$, $E = \frac{-13.6}{9} = -1.5\ \text{eV}$

This energy is approximately equal to the energy of gaseous hydrogen. It can be concluded that the electron has jumped from $n = 1$ to $n = 3$ level.

During its de-excitation, the electrons can jump from $n = 3$ to $n = 1$ directly, which forms a line of the Lyman series of the hydrogen spectrum.

We have the relation for wave number for Lyman series as:

$$\frac{1}{\lambda} = R_y\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

Where,

R_y = Rydberg constant = 1.097×10^7 m^{-1}

λ= Wavelength of radiation emitted by the transition of the electron

For n = 3, we can obtain λas:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 1.097\times10^7\left(1 - \frac{1}{9}\right) = 1.097\times10^7\times\frac{8}{9}$$

$$\lambda = \frac{9}{8\times1.097\times10^7} = 102.55\ nm$$

If the electron jumps from n = 2 to n = 1, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 1.097\times10^7\left(1 - \frac{1}{4}\right) = 1.097\times10^7\times\frac{3}{4}$$

$$\lambda = \frac{4}{1.097\times10^7\times3} = 121.54\ nm$$

If the transition takes place from n = 3 to n = 2, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 1.097\times10^7\left(\frac{1}{4} - \frac{1}{9}\right) = 1.097\times10^7\times\frac{5}{36}$$

$$\lambda = \frac{36}{5\times1.097\times10^7} = 656.33\ nm$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.

Hence, in Lyman series, two wavelengths i.e., 102.5 nm and 121.5 nm are emitted. And in the Balmer series, one wavelength i.e., 656.33 nm is emitted.

Question 12.10:

In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)

Answer

Radius of the orbit of the Earth around the Sun, $r = 1.5 \times 10^{11}$ m

Orbital speed of the Earth, $v = 3 \times 10^4$ m/s

Mass of the Earth, $m = 6.0 \times 10^{24}$ kg

According to Bohr's model, angular momentum is quantized and given as:

$$mvr = \frac{nh}{2\pi}$$

Where,

h = Planck's constant = 6.62×10^{-34} Js

n = Quantum number

$$\therefore n = \frac{mvr2\pi}{h}$$

$$= \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11}}{6.62 \times 10^{-34}}$$

$$= 25.61 \times 10^{73} = 2.6 \times 10^{74}$$

Hence, the quanta number that characterizes the Earth' revolution is 2.6×10^{74}.

Question 12.11:

Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Answer

(a) about the same

The average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model is about the same size as predicted by Rutherford's model. This is because the average angle was taken in both models.

(b) much less

The probability of scattering of α-particles at angles greater than 90° predicted by Thomson's model is much less than that predicted by Rutherford's model.

(c) Scattering is mainly due to single collisions. The chances of a single collision increases linearly with the number of target atoms. Since the number of target atoms increase with an increase in thickness, the collision probability depends linearly on the thickness of the target.

(d) Thomson's model

It is wrong to ignore multiple scattering in Thomson's model for the calculation of average angle of scattering of α−particles by a thin foil. This is because a single collision causes very little deflection in this model. Hence, the observed average scattering angle can be explained only by considering multiple scattering.

Question 12.12:

The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Answer

Radius of the first Bohr orbit is given by the relation,

$$r_1 = \frac{4\pi \epsilon_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} \quad \ldots (1)$$

Where,

ϵ_0 = Permittivity of free space

h = Planck's constant = 6.63×10^{-34} Js

m_e = Mass of an electron = 9.1×10^{-31} kg

e = Charge of an electron = 1.9×10^{-19} C

m_p = Mass of a proton = 1.67×10^{-27} kg

r = Distance between the electron and the proton

Coulomb attraction between an electron and a proton is given as:

$$F_C = \frac{e^2}{4\pi \epsilon_0 r^2} \quad \ldots (2)$$

Gravitational force of attraction between an electron and a proton is given as:

$$F_G = \frac{Gm_p m_e}{r^2} \quad \ldots (3)$$

Where,

G = Gravitational constant = 6.67×10^{-11} N m^2/kg^2

If the electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equal, then we can write:

$\therefore F_G = F_C$

$$\frac{Gm_p m_e}{r^2} = \frac{e^2}{4\pi \epsilon_0 r^2}$$

$$\therefore \frac{e^2}{4\pi \epsilon_0} = Gm_p m_e \quad \ldots (4)$$

Putting the value of equation (4) in equation (1), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_e^2}$$

$$= \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times\left(9.1\times10^{-31}\right)^2} \approx 1.21\times10^{29}\ m$$

It is known that the universe is 156 billion light years wide or 1.5 × 10^{27} m wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

Question 12.13:

Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level *n* to level (*n*−1). For large *n*, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Answer

It is given that a hydrogen atom de-excites from an upper level (*n)* to a lower level (*n*−1).

We have the relation for energy (E_1) of radiation at level *n* as:

$$E_1 = h\nu_1 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \qquad \ldots \text{(i)}$$

Where,

ν_1 = Frequency of radiation at level n

h = Planck's constant

m = Mass of hydrogen atom

e = Charge on an electron

$\in_0$ = Permittivity of free space

Now, the relation for energy (E_2) of radiation at level ($n - 1$) is givenas:

$$E_2 = h\nu_2 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \quad \dots (ii)$$

Where,

ν_2 = Frequency of radiation at level $(n-1)$

Energy (E) released as a result of de-excitation:

$E = E_2 - E_1$

$hv = E_2 - E_1$ … (iii)

Where,

v = Frequency of radiation emitted

Putting values from equations (i) and (ii) in equation (iii), we get:

$$\nu = \frac{me^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4(2n-1)}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \simeq 2n$ and $(n-1) \simeq n$.

$$\therefore \nu = \frac{me^4}{32\pi^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \dots (iv)$$

Classical relation of frequency of revolution of an electron is given as:

$$\nu_c = \frac{v}{2\pi r} \quad \dots (v)$$

Where,

Velocity of the electron in the n^{th} orbit is given as:

$$v = \frac{e^2}{4\pi \in_0 \left(\frac{h}{2\pi}\right) n} \quad \dots (vi)$$

And, radius of the n^{th} orbit is given as:

$$r = \frac{4\pi \epsilon_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \qquad \text{... (vii)}$$

Putting the values of equations (vi) and (vii) in equation (v), we get:

$$v_c = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \qquad \text{... (viii)}$$

Hence, the frequency of radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

Question 12.14:

Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10} m).

(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.

Answer

(a) Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Speed of light, $c = 3 \times 10^{8}$ m/s

Let us take a quantity involving the given quantities as $\left(\frac{e^2}{4\pi \in_0 m_e c^2}\right)$.

Where,

$\in_0$ = Permittivity of free space

And, $\frac{1}{4\pi \in_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \in_0}\times\frac{e^2}{m_e c^2}$$

$$=9\times10^9\times\frac{\left(1.6\times10^{-19}\right)^2}{9.1\times10^{-31}\times\left(3\times10^8\right)^2}$$

$$=2.81\times10^{-15}\text{ m}$$

Hence, the numerical value of the taken quantity is much smaller than the typical size of an atom.

(b) Charge on an electron, e = 1.6 × 10^{-19} C

Mass of an electron, m_e = 9.1 × 10^{-31} kg

Planck's constant, h = 6.63 ×10^{-34} Js

Let us take a quantity involving the given quantities as $\frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2}$.

Where,

$\in_0$ = Permittivity of free space

And, $\frac{1}{4\pi \in_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$

The numerical value of the taken quantity will be:

$$4\pi \in_0 \times \frac{\left(\frac{h}{2\pi}\right)^2}{m_e e^2}$$

$$= \frac{1}{9\times10^9} \times \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$$

$$= 0.53\times10^{-10}\ m$$

Hence, the value of the quantity taken is of the order of the atomic size.

Question 12.15:

The total energy of an electron in the first excited state of the hydrogen atom is about −3.4 eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Answer

(a) Total energy of the electron, $E = -3.4$ eV

Kinetic energy of the electron is equal to the negative of the total energy.

$\Rightarrow K = -E$

= − (− 3.4) = +3.4 eV

Hence, the kinetic energy of the electron in the given state is +3.4 eV.

(b) Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

$\Rightarrow U = -2\ K$

= − 2 × 3.4 = − 6.8 eV

Hence, the potential energy of the electron in the given state is − 6.8 eV.

(c) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since total energy is the sum of kinetic and potential energies, total energy of the system will also change.

Question 12.16:

If Bohr's quantisation postulate (angular momentum = *nh*/2π) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Answer

We never speak of quantization of orbits of planets around the Sun because the angular momentum associated with planetary motion is largely relative to the value of Planck's constant *(h)*. The angular momentum of the Earth in its orbit is of the order of $10^{70}h$. This leads to a very high value of quantum levels *n* of the order of 10^{70}. For large values of *n*, successive energies and angular momenta are relatively very small. Hence, the quantum levels for planetary motion are considered continuous.

Question 12.17:

Obtain the first Bohr's radius and the ground state energy of a *muonic hydrogen atom* [i.e., an atom in which a negatively charged muon (μ^-) of mass about $207m_e$ orbits around a proton].

Answer

Mass of a negatively charged muon, $m_\mu = 207m_e$

According to Bohr's model,

Bohr radius, $r_e \propto \left(\frac{1}{m_e}\right)$

And, energy of a ground state *electronic hydrogen atom*, $E_e \propto m_e$.

Also, energy of a ground state *muonic hydrogen atom*, $E_\mu \propto m_\mu$.

We have the value of the first Bohr orbit, $r_e = 0.53 \text{ Å} = 0.53 \times 10^{-10}$ m

Let r_μ be the radius of *muonic hydrogen atom*.

At equilibrium, we can write the relation as:

$$m_\mu r_\mu = m_e r_e$$

$$207 m_e \times r_\mu = m_e r_e$$

$$\therefore r_\mu = \frac{0.53\times10^{-10}}{207} = 2.56\times10^{-13} \text{ m}$$

Hence, the value of the first Bohr radius of a *muonic hydrogen atom* is 2.56×10^{-13} m.

We have,

$E_e = -13.6$ eV

Take the ratio of these energies as:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e}$$

$$E_\mu = 207 E_e$$

$$= 207\times(-13.6) = -2.81 \text{ keV}$$

Hence, the ground state energy of a *muonic hydrogen atom* is −2.81 keV.

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Question 13.1:

(a) Two stable isotopes of lithium ${}^{6}_{3}\mathrm{Li}$ and ${}^{7}_{3}\mathrm{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, ${}^{10}_{5}\mathrm{B}$ and ${}^{11}_{5}\mathrm{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_{5}\mathrm{B}$ and ${}^{11}_{5}\mathrm{B}$.

Answer

(a) Mass of lithium isotope ${}^{6}_{3}\mathrm{Li}$, m_1 = 6.01512 u

Mass of lithium isotope ${}^{7}_{3}\mathrm{Li}$, m_2 = 7.01600 u

Abundance of ${}^{6}_{3}\mathrm{Li}$, η_1= 7.5%

Abundance of ${}^{7}_{3}\mathrm{Li}$, η_2= 92.5%

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{92.5 + 7.5}$$

$$= 6.940934 \text{ u}$$

(b) Mass of boron isotope ${}^{10}_{5}\mathrm{B}$, m_1 = 10.01294 u

Mass of boron isotope ${}^{11}_{5}\mathrm{B}$, m_2 = 11.00931 u

Abundance of ${}^{10}_{5}\mathrm{B}$, η_1 = x%

Abundance of ${}^{11}_{5}\mathrm{B}$, η_2= (100 − x)%

Atomic mass of boron, m = 10.811 u

The atomic mass of boron atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$1081.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\therefore x = \frac{19.821}{0.99637} = 19.89\%$$

And 100 − x = 80.11%

Hence, the abundance of ${}^{10}_{5}\mathrm{B}$ is 19.89% and that of ${}^{11}_{5}\mathrm{B}$ is 80.11%.

Question 13.2:

The three stable isotopes of neon: ${}^{20}_{10}\mathrm{Ne}, {}^{21}_{10}\mathrm{Ne}$ and ${}^{22}_{10}\mathrm{Ne}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Answer

Atomic mass of ${}^{20}_{10}\mathrm{Ne}$, m_1= 19.99 u

Abundance of ${}^{20}_{10}\mathrm{Ne}$, η_1 = 90.51%

Atomic mass of ${}^{21}_{10}\mathrm{Ne}$, m_2 = 20.99 u

Abundance of ${}^{21}_{10}\mathrm{Ne}$, η_2 = 0.27%

Atomic mass of ${}^{22}_{10}\mathrm{Ne}$, m_3 = 21.99 u

Abundance of ${}^{22}_{10}\mathrm{Ne}$, η_3 = 9.22%

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$$= 20.1771 \text{ u}$$

Question 13.3:

Obtain the binding energy (in MeV) of a nitrogen nucleus $\left({}^{14}_{7}\mathrm{N}\right)$, given $m\left({}^{14}_{7}\mathrm{N}\right)$=14.00307 u

Answer

Atomic mass of nitrogen $\left({}_{7}\mathrm{N}^{14}\right)$, m = 14.00307 u

A nucleus of nitrogen ${}_{7}\mathrm{N}^{14}$ contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n= 1.008665 u

$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

= 7.054775 + 7.06055 − 14.00307

= 0.11236 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.11236 \times 931.5$ MeV/c^2

Hence, the binding energy of the nucleus is given as:

$E_b = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_b = 0.11236 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 104.66334 MeV

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

Question 13.4:

Obtain the binding energy of the nuclei $^{56}_{26}\text{Fe}$ and $^{209}_{83}\text{Bi}$ in units of MeV from the following data:

$m\left(^{56}_{26}\text{Fe}\right)$ = 55.934939 u $m\left(^{209}_{83}\text{Bi}\right)$ = 208.980388 u

Answer

Atomic mass of $^{56}_{26}\text{Fe}$, m_1 = 55.934939 u

$^{56}_{26}\text{Fe}$ nucleus has 26 protons and (56 − 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

= 26.20345 + 30.25995 − 55.934939

= 0.528461 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.528461 \times 931.5$ MeV/c^2

The binding energy of this nucleus is given as:

$E_{b1} = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 492.26 MeV

Average binding energy per nucleon $= \frac{492.26}{56} = 8.79 \text{ MeV}$

Atomic mass of $^{209}_{83}\text{Bi}$, m_2 = 208.980388 u

$^{209}_{83}\text{Bi}$ nucleus has 83 protons and (209 − 83) 126 neutrons.

Hence, the mass defect of this nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

= 83.649475 + 127.091790 − 208.980388

= 1.760877 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m' = 1.760877 \times 931.5 \text{ MeV}/c^2$

Hence, the binding energy of this nucleus is given as:

$E_{b2} = \Delta m' c^2$

$= 1.760877 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

$= 1640.26 \text{ MeV}$

Average bindingenergy per nucleon = $\frac{1640.26}{209} = 7.848 \text{ MeV}$

Question 13.5:

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u).

Answer

Mass of a copper coin, m' = 3 g

Atomic mass of ${}_{29}\text{Cu}^{63}$ atom, m = 62.92960 u

The total number of ${}_{29}\text{Cu}^{63}$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where,

N_A = Avogadro's number = 6.023×10^{23} atoms /g

Mass number = 63 g

$\therefore N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22} \text{ atoms}$

${}_{29}\text{Cu}^{63}$ nucleus has 29 protons and (63 − 29) 34 neutrons

$\therefore$Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

$= 0.591935$ u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2

Hence, the binding energy of the nuclei of the coin is given as:

$E_b = \Delta mc^2$

$= 1.69766958 \times 10^{22} \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

$= 1.581 \times 10^{25}$ MeV

But 1 MeV $= 1.6 \times 10^{-13}$ J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$= 2.5296 \times 10^{12}$ J

This much energy is required to separate all the neutrons and protons from the given coin.

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Question 13.6:

Write nuclear reaction equations for

(i) *α*-decay of $^{226}_{88}\text{Ra}$ **(ii)** *α*-decay of $^{242}_{94}\text{Pu}$

(iii) β^--decay of $^{32}_{15}\text{P}$ **(iv)** β^--decay of $^{210}_{83}\text{Bi}$

(v) β^+-decay of $^{11}_{6}\text{C}$ **(vi)** β^+-decay of $^{97}_{43}\text{Tc}$

(vii) Electron capture of $^{120}_{54}\text{Xe}$

Answer

α is a nucleus of helium $\left({}_2\text{He}^4\right)$ and β is an electron (e^- for β^- and e^+ for β^+). In every *α*-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

(i) $${}_{88}\text{Ra}^{226} \longrightarrow {}_{86}\text{Rn}^{222} + {}_2\text{He}^4$$

(ii) $$^{242}_{94}\text{Pu} \longrightarrow {}^{238}_{92}\text{U} + {}^4_2\text{He}$$

(iii) $$^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^- + \overline{\nu}$$

(iv) $$^{210}_{83}\text{B} \longrightarrow {}^{210}_{84}\text{PO} + e^- + \overline{\nu}$$

(v) $$^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + e^+ + \nu$$

(vi) $$^{97}_{43}\text{Tc} \longrightarrow {}^{97}_{42}\text{MO} + e^+ + \nu$$

(vii) $$^{120}_{54}\text{Xe} + e^+ \longrightarrow {}^{120}_{53}\text{I} + \nu$$

Question 13.7:

A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Answer

Half-life of the radioactive isotope = T years

Original amount of the radioactive isotope = N_0

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

Where,

λ = Decay constant

t = Time

$$\therefore -\lambda t = \frac{1}{32}$$

$$-\lambda t = \ln 1 - \ln 32$$
$$-\lambda t = 0 - 3.4657$$

$$t = \frac{3.4657}{\lambda}$$

Since $\lambda = \frac{0.693}{T}$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T \text{ years}$$

Hence, the isotope will take about $5T$ years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$

$$-\lambda t = \ln 1 - \ln 100$$

$$-\lambda t = 0 - 4.6052$$

$$t = \frac{4.6052}{\lambda}$$

Since, $\lambda = 0.693/T$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645T \text{ years}$$

Hence, the isotope will take about 6.645*T* years to reduce to 1% of its original value.

Question 13.8:

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Answer

Decay rate of living carbon-containing matter, R = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of ${}^{14}_{6}C$, $T_{1/2}$ = 5730 years

The decay rate of the specimen obtained from the Mohenjodaro site:

R' = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, $\lambda$and time, t as:

$$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$-\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\therefore t = \frac{0.5108}{\lambda}$$

$$\text{But } \lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{5730}$$

$$\therefore t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5 \text{ years}$$

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 years.

Question 13.9:

Obtain the amount of $^{60}_{27}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}\text{Co}$ is 5.3 years.

Answer

The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$

$$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$$

$$= 29.6 \times 10^{7} \text{ decay/s}$$

Where,

N = Required number of atoms

Half-life of $^{60}_{27}\text{Co}$, $T_{\frac{1}{2}}$ = 5.3 years

$= 5.3 \times 365 \times 24 \times 60 \times 60$

$= 1.67 \times 10^8$ s

For decay constant λ, we have the rate of decay as:

$$\frac{dN}{dt} = \lambda N$$

Where, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^8}\ \text{s}^{-1}$

$$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$$

$$= \frac{29.6 \times 10^7}{\frac{0.693}{1.67 \times 10^8}} = 7.133 \times 10^{16}\ \text{atoms}$$

For ${}_{27}\text{Co}^{60}$:

Mass of 6.023×10^{23} (Avogadro's number) atoms = 60 g

$\therefore$Mass of 7.133×10^{16} atoms $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6}\ \text{g}$

Hence, the amount of ${}_{27}\text{Co}^{60}$ necessary for the purpose is 7.106×10^{-6} g.

Question 13.10:

The half-life of ${}^{90}_{38}\text{Sr}$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Answer

Half life of ${}^{90}_{38}\text{Sr}$, $t_{1/2}$ = 28 years

$= 28 \times 365 \times 24 \times 60 \times 60$

$= 8.83 \times 10^8$ s

Mass of the isotope, m = 15 mg

90 g of ${}^{90}_{38}\text{Sr}$ atom contains 6.023×10^{23} (Avogadro's number) atoms.

Therefore, 15 mg of ${}^{90}_{38}\mathrm{Sr}$ contains:

$$\frac{6.023\times10^{23}\times15\times10^{-3}}{90}\text{, i.e., }1.0038\times10^{20}\text{ number of atoms}$$

Rate of disintegration, $\frac{dN}{dt}=\lambda N$

Where,

λ = Decay constant $=\frac{0.693}{8.83\times10^{8}}\ \mathrm{s}^{-1}$

$$\therefore\frac{dN}{dt}=\frac{0.693\times1.0038\times10^{20}}{8.83\times10^{8}}=7.878\times10^{10}\text{ atoms/s}$$

Hence, the disintegration rate of 15 mg of the given isotope is

7.878×10^{10} atoms/s.

Question 13.11:

Obtain approximately the ratio of the nuclear radii of the gold isotope ${}^{197}_{79}\mathrm{Au}$ and the silver isotope ${}^{107}_{47}\mathrm{Ag}$.

Answer

Nuclear radius of the gold isotope ${}_{79}\mathrm{Au}^{197}$ = R_{Au}

Nuclear radius of the silver isotope ${}_{47}\mathrm{Ag}^{107}$ = R_{Ag}

Mass number of gold, A_{Au} = 197

Mass number of silver, A_{Ag} = 107

The ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}}=\left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}}$$

$$=\left(\frac{197}{107}\right)^{\frac{1}{3}}=1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

Question 13.12:

Find the Q-value and the kinetic energy of the emitted *α*-particle in the α-decay of (a) ${}^{226}_{88}\mathrm{Ra}$ and (b) ${}^{220}_{86}\mathrm{Rn}$.

Given $m\left({}^{226}_{88}\mathrm{Ra}\right)$ = 226.02540 u, $m\left({}^{222}_{86}\mathrm{Rn}\right)$ = 222.01750 u,

$m\left({}^{220}_{86}\mathrm{Rn}\right)$ = 220.01137 u, $m\left({}^{216}_{84}\mathrm{Po}\right)$ = 216.00189 u.

Answer

(a) Alpha particle decay of ${}^{226}_{88}\mathrm{Ra}$ emits a helium nucleus. As a result, its mass number reduces to (226 − 4) 222 and its atomic number reduces to (88 − 2) 86. This is shown in the following nuclear reaction.

$${}^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Ra} + {}^{4}_{2}\mathrm{He}$$

Q-value of

emitted *α*-particle = (Sum of initial mass − Sum of final mass) c^2

Where,

c = Speed of light

It is given that:

$$m\left({}^{226}_{88}\mathrm{Ra}\right) = 226.02540 \text{ u}$$

$$m\left({}^{222}_{86}\mathrm{Rn}\right) = 222.01750 \text{ u}$$

$$m\left({}^{4}_{2}\mathrm{He}\right) = 4.002603 \text{ u}$$

Q-value = [226.02540 − (222.01750 + 4.002603)] u c^2

= 0.005297 u c^2

But 1 u = 931.5 MeV/c^2

∴Q = 0.005297 × 931.5 ≈ 4.94 MeV

Kinetic energy of the *α*-particle $= \left(\dfrac{\text{Mass number after decay}}{\text{Mass number before decay}}\right) \times Q$

$$= \frac{222}{226} \times 4.94 = 4.85 \text{ MeV}$$

(b) Alpha particle decay of $\left({}^{220}_{86}\text{Rn}\right)$ is shown by the following nuclear reaction.

$${}^{220}_{86}\text{Rn} \longrightarrow {}^{216}_{84}\text{Po} + {}^{4}_{2}\text{He}$$

It is given that:

Mass of $\left({}^{220}_{86}\text{Rn}\right)$ = 220.01137 u

Mass of $\left({}^{216}_{84}\text{Po}\right)$ = 216.00189 u

$\therefore Q$-value = $\left[220.01137 - (216.00189 + 4.00260)\right] \times 931.5$

≈ 641 MeV

Kinetic energy of the α-particle $= \left(\frac{220 - 4}{220}\right) \times 6.41$

= 6.29 MeV

Question 13.13:

The radionuclide ^{11}C decays according to

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \nu: \quad T_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$$m\left({}^{11}_{6}\text{C}\right) = 11.011434 \text{ u and } m\left({}^{11}_{6}\text{B}\right) = 11.009305 \text{ u,}$$

calculate Q and compare it with the maximum energy of the positron emitted

Answer

The given nuclear reaction is:

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \nu$$

Half life of ${}^{11}_{6}$C nuclei, $T_{1/2} = 20.3$ min

Atomic mass of $m\left({}^{11}_{6}\mathrm{C}\right)$ = 11.011434 u

Atomic mass of $m\left({}^{11}_{6}\mathrm{B}\right) = 11.009305$ u

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the ${}^{11}_{6}\mathrm{C}$ nucleus is given as:

$$\Delta Q = \left[m'\left({}_{6}\mathrm{C}^{11}\right) - \left[m'\left({}^{11}_{5}\mathrm{B}\right) + m_e\right]\right]c^2 \quad \ldots (1)$$

Where,

m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light

m' = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the case of ${}^{11}\mathrm{C}$ and 5 m_e in the case of ${}^{11}\mathrm{B}$.

Hence, equation (1) reduces to:

$$\Delta Q = \left[m\left({}_{6}\mathrm{C}^{11}\right) - m\left({}^{11}_{5}\mathrm{B}\right) - 2m_e\right]c^2$$

Here, $m\left({}_{6}\mathrm{C}^{11}\right)$ and $m\left({}^{11}_{5}\mathrm{B}\right)$ are the atomic masses.

$\therefore \Delta Q = [11.011434 - 11.009305 - 2 \times 0.000548]\, c^2$

$= (0.001033\, c^2)$ u

But 1 u = 931.5 Mev/c^2

$\therefore \Delta Q = 0.001033 \times 931.5 \approx 0.962$ MeV

The value of Q is almost comparable to the maximum energy of the emitted positron.

Question 13.14:

The nucleus ${}^{23}_{10}\mathrm{Ne}$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\left({}^{23}_{10}\mathrm{Ne}\right)$ = 22.994466 u

$m\left({}^{23}_{11}\mathrm{Na}\right)$ = 22.989770 u.

Answer

In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus ${}^{23}_{10}\mathrm{Ne}$ is given as:

$${}^{23}_{10}\mathrm{Ne} \rightarrow {}^{23}_{11}\mathrm{Na} + e^- + \bar{\nu} + Q$$

It is given that:

Atomic mass of $m\left({}^{23}_{10}\mathrm{Ne}\right)$ = 22.994466 u

Atomic mass of $m\left({}^{23}_{11}\mathrm{Na}\right)$ = 22.989770 u

Mass of an electron, m_e = 0.000548 u

Q-value of the given reaction is given as:

$$Q = \left[m\left({}^{23}_{10}\mathrm{Ne}\right) - \left[m\left({}^{23}_{11}\mathrm{Na}\right) + m_e\right]\right]c^2$$

There are 10 electrons in ${}^{23}_{10}\mathrm{Ne}$ and 11 electrons in ${}^{23}_{11}\mathrm{Na}$. Hence, the mass of the electron is cancelled in the Q-value equation.

$$\therefore Q = [22.994466 - 22.989770]c^2$$
$$= \left(0.004696\ c^2\right)\ \mathrm{u}$$

$$\text{But } 1\ \mathrm{u} = 931.5\ \mathrm{MeV}/c^2$$

$$\therefore Q = 0.004696 \times 931.5 = 4.374\ \mathrm{MeV}$$

The daughter nucleus is too heavy as compared to e^- and $\bar{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the *Q*-value, i.e., 4.374 MeV.

Question 13.15:

The *Q* value of a nuclear reaction *A* + *b* → *C* + *d* is defined by

$Q = [\ m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the *Q*-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}_1^1\mathrm{H} + {}_1^3\mathrm{H} \rightarrow {}_1^2\mathrm{H} + {}_1^2\mathrm{H}$

(ii) ${}_6^{12}\mathrm{C} + {}_6^{12}\mathrm{C} \rightarrow {}_{10}^{20}\mathrm{Ne} + {}_2^4\mathrm{He}$

Atomic masses are given to be

$m\left({}_1^2\mathrm{H}\right) = 2.014102\ \mathrm{u}$

$m\left({}_1^3\mathrm{H}\right) = 3.016049\ \mathrm{u}$

$m\left({}_6^{12}\mathrm{C}\right) = 12.000000\,\mathrm{u}$

$m\left({}_{10}^{20}\mathrm{Ne}\right) = 19.992439\,\mathrm{u}$

Answer

(i) The given nuclear reaction is:

${}_1\mathrm{H}^1 + {}_1^3\mathrm{H} \rightarrow {}_1^2\mathrm{H} + {}_1^2\mathrm{H}$

It is given that:

Atomic mass $m\left({}_1^1\mathrm{H}\right) = 1.007825\ \mathrm{u}$

Atomic mass $m\left({}_1^3\mathrm{H}\right) = 3.016049\ \mathrm{u}$

Atomic mass $m\left({}_1^2\mathrm{H}\right) = 2.014102\ \mathrm{u}$

According to the question, the *Q*-value of the reaction can be written as:

$$Q=\left[m\left({}_1^1\text{H}\right)+m\left({}_1^3\text{H}\right)-2m\left({}_1^2\text{H}\right)\right]c^2$$

$$=\left[1.007825+3.016049-2\times 2.014102\right]c^2$$

$$Q=\left(-0.00433\ c^2\right)\ \text{u}$$

$$\text{But}\ \ 1\ \text{u}=931.5\ \text{MeV}/c^2$$

$$\therefore Q=-0.00433\times 931.5=-4.0334\ \text{MeV}$$

The negativeQ-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

$${}_6^{12}\text{C}+{}_6^{12}\text{C}\rightarrow {}_{10}^{20}\text{Ne}+{}_2^4\text{He}$$

It is given that:

Atomic mass of $m\left({}_6^{12}\text{C}\right)=12.0\ \text{u}$

Atomic mass of $m\left({}_{10}^{20}\text{Ne}\right)=19.992439\ \text{u}$

Atomic mass of $m\left({}_2^4\text{He}\right)=4.002603\ \text{u}$

The *Q*-value of this reaction is given as:

$$Q=\left[2m\left({}_6^{12}\text{C}\right)-m\left({}_{10}^{20}\text{Ne}\right)-m\left({}_2^4\text{He}\right)\right]c^2$$

$$=\left[2\times 12.0-19.992439-4.002603\right]c^2$$

$$=\left(0.004958\ c^2\right)\ \text{u}$$

$$=0.004958\times 931.5=4.618377\ \text{MeV}$$

The positive *Q*-value of the reaction shows that the reaction is exothermic.

Question 13.16:

Suppose, we think of fission of a ${}^{56}_{26}\mathrm{Fe}$ nucleus into two equal fragments, ${}^{28}_{13}\mathrm{Al}$. Is the fission energetically possible? Argue by working out *Q* of the process. Given $m\left({}^{56}_{26}\mathrm{Fe}\right) = 55.93494\ \mathrm{u}$ and $m\left({}^{28}_{13}\mathrm{Al}\right) = 27.98191\,\mathrm{u}$.

Answer

The fission of ${}^{56}_{26}\mathrm{Fe}$ can be given as:

$${}^{56}_{13}\mathrm{Fe} \longrightarrow 2\,{}^{28}_{13}\mathrm{Al}$$

It is given that:

Atomic mass of $m\left({}^{56}_{26}\mathrm{Fe}\right)$ = 55.93494 u

Atomic mass of $m\left({}^{28}_{13}\mathrm{Al}\right) = 27.98191\ \mathrm{u}$

The *Q*-value of this nuclear reaction is given as:

$$Q = \left[m\left({}^{56}_{26}\mathrm{Fe}\right) - 2m\left({}^{28}_{13}\mathrm{Al}\right)\right]c^2$$

$$= \left[55.93494 - 2 \times 27.98191\right]c^2$$

$$= \left(-0.02888\ c^2\right)\ \mathrm{u}$$

$$\text{But } 1\,\mathrm{u} = 931.5\ \mathrm{MeV}/c^2$$

$$\therefore Q = -0.02888 \times 931.5 = -26.902\ \mathrm{MeV}$$

The *Q*-value of the fission is negative. Therefore, the fission is not possible energetically. For an energetically-possible fission reaction, the *Q*-value must be positive.

Question 13.17:

The fission properties of ${}^{239}_{94}\mathrm{Pu}$ are very similar to those of ${}^{235}_{92}\mathrm{U}$.

The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}\mathrm{Pu}$ undergo fission?

Answer

Average energy released per fission of ${}^{239}_{94}\mathrm{Pu}$, $E_{av} = 180\ \mathrm{MeV}$

Amount of pure ${}_{94}\mathrm{Pu}^{239}$, *m* = 1 kg = 1000 g

N_A= Avogadro number = 6.023 × 10^{23}

Mass number of ${}^{239}_{94}\mathrm{Pu}$ = 239 g

1 mole of ${}_{94}\mathrm{Pu}^{239}$ contains N_A atoms.

∴m g of ${}_{94}\mathrm{Pu}^{239}$ contains $\left(\frac{N_A}{\text{Mass number}} \times m\right)$atoms

$$= \frac{6.023 \times 10^{23}}{239} \times 1000 = 2.52 \times 10^{24} \text{ atoms}$$

∴Total energy released during the fission of 1 kg of ${}^{239}_{94}\mathrm{Pu}$ is calculated as:

$$E = E_{av} \times 2.52 \times 10^{24}$$
$$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26} \text{ MeV}$$

Hence, 4.536×10^{26} MeV is released if all the atoms in 1 kg of pure ${}_{94}\mathrm{Pu}^{239}$ undergo fission.

Question 13.18:

A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}^{235}_{92}\mathrm{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}^{235}_{92}\mathrm{U}$ and that this nuclide is consumed only by the fission process.

Answer

Half life of the fuel of the fission reactor, $t_{\frac{1}{2}} = 5$ years

= 5 × 365 × 24 × 60 × 60 s

We know that in the fission of 1 g of $^{235}_{92}\text{U}$ nucleus, the energy released is equal to 200 MeV.

1 mole, i.e., 235 g of $^{235}_{92}\text{U}$ contains 6.023 × 10^{23} atoms.

∴1 g $^{235}_{92}\text{U}$ contains $\frac{6.023 \times 10^{23}}{235}$ atoms

The total energy generated per gram of $^{235}_{92}\text{U}$ is calculated as:

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV/g}$$
$$= \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{6}}{235} = 8.20 \times 10^{10} \text{ J/g}$$

The reactor operates only 80% of the time.

Hence, the amount of $^{235}_{92}\text{U}$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$= \frac{5 \times 80 \times 60 \times 60 \times 365 \times 24 \times 1000 \times 10^{6}}{100 \times 8.20 \times 10^{10}} \text{ g}$$
$$\approx 1538 \text{ kg}$$

∴Initial amount of $^{235}_{92}\text{U}$ = 2 × 1538 = 3076 kg

Question 13.19:

How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \longrightarrow ^{3}_{1}\text{He} + \text{n} + 3.27 \text{ MeV}$$

Answer

The given fusion reaction is:

$${}_1^2\text{H} + {}_1^2\text{H} \longrightarrow {}_2^3\text{He} + \text{n} + 3.27 \text{ MeV}$$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

∴2.0 kg of deuterium contains $= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

∴Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$= 1.576 \times 10^{14} \text{ J}$$

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576 \times 10^{14}}{100} \text{ s}$$

$$\frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365} \approx 4.9 \times 10^{4} \text{ years}$$

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Question 13.20:

Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Answer

When two deuterons collide head-on, the distance between their centres, *d* is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2×10^{-15} m

$\therefore d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}$ m

Charge on a deuteron nucleus = Charge on an electron = $e = 1.6 \times 10^{-19}$ C

Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi \epsilon_0 d}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} \text{ J}$$

$$= \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15} \times \left(1.6 \times 10^{-19}\right)} \text{ eV}$$

$$= 360 \text{ keV}$$

Hence, the height of the potential barrier of the two-deuteron system is

360 keV.

Question 13.21:

From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Answer

We have the expression for nuclear radius as:

$R = R_0A^{1/3}$

Where,

R_0 = Constant.

A = Mass number of the nucleus

Nuclear matter density, $\rho = \dfrac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$

Let m be the average mass of the nucleus.

Hence, mass of the nucleus = mA

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.

Question 13.22:

For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K−shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}^A_Z X \rightarrow {}^A_{Z-1} Y + \nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice−versa.

Answer

Let the amount of energy released during the electron capture process be Q_1. The nuclear reaction can be written as:

$$e^{+} + {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + \nu + Q_1 \qquad \ldots (1)$$

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction can be written as:

$${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + \nu + Q_2 \qquad \ldots (2)$$

$m_N\left({}_{Z}^{A}X\right)$ = Nuclear mass of ${}_{Z}^{A}X$

$m_N\left({}_{Z-1}^{A}Y\right)$ = Nuclear mass of ${}_{Z-1}^{A}Y$

$m\left({}_{Z}^{A}X\right)$ = Atomic mass of ${}_{Z}^{A}X$

$m\left({}_{Z-1}^{A}Y\right)$ = Atomic mass of ${}_{Z-1}^{A}Y$

m_e = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

$$Q_1 = \left[m_N\left({}_{Z}^{A}X\right) + m_e - m_N\left({}_{Z-1}^{A}Y\right)\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - Zm_e + m_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right)\right]c^2 \qquad \ldots (3)$$

Q-value of the positron capture reaction is given as:

$$Q_2 = \left[m_N\left({}_{Z}^{A}X\right) - m_N\left({}_{Z-1}^{A}Y\right) - m_e\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - Zm_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right) - 2m_e\right]c^2 \qquad \ldots (4)$$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

In other words, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the *Q*-value must be positive for an energetically-allowed nuclear reaction.

Question 13.23:

In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}^{24}_{12}\mathrm{Mg}$ (23.98504u), ${}^{25}_{12}\mathrm{Mg}$ (24.98584u) and ${}^{26}_{12}\mathrm{Mg}$ (25.98259u). The natural abundance of ${}^{24}_{12}\mathrm{Mg}$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Answer

Average atomic mass of magnesium, m = 24.312 u

Mass of magnesium isotope ${}^{24}_{12}\mathrm{Mg}$, m_1 = 23.98504 u

Mass of magnesium isotope ${}^{25}_{12}\mathrm{Mg}$, m_2 = 24.98584 u

Mass of magnesium isotope ${}^{26}_{12}\mathrm{Mg}$, m_3 = 25.98259 u

Abundance of ${}^{24}_{12}\mathrm{Mg}$, η_1= 78.99%

Abundance of ${}^{25}_{12}\mathrm{Mg}$, $\eta_2 = x\%$

Hence, abundance of ${}^{26}_{12}\mathrm{Mg}$, $\eta_3 = 100 - x - 78.99\% = (21.01 - x)\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$2431.2 = 1894.5783096 + 24.98584x + 545.8942159 - 25.98259x$$

$$0.99675x = 9.2725255$$

$$\therefore x \approx 9.3\%$$

$$\text{And } 21.01 - x = 11.71\%$$

Hence, the abundance of ${}^{25}_{12}\mathrm{Mg}$ is 9.3% and that of ${}^{26}_{12}\mathrm{Mg}$ is 11.71%.

Question 13.24:

The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}\mathrm{Ca}$ and ${}^{27}_{13}\mathrm{Al}$ from the following data:

$m\left({}^{40}_{20}\mathrm{Ca}\right)$ = 39.962591 u

$m\left({}^{41}_{20}\mathrm{Ca}\right)$) = 40.962278 u

$m\left({}^{26}_{13}\mathrm{Al}\right)$ = 25.986895 u

$m\left({}^{27}_{13}\mathrm{Al}\right)$) = 26.981541 u

Answer

For ${}^{41}_{20}\mathrm{Ca}$: Separation energy $= 8.363007$ MeV

For ${}^{27}_{13}\mathrm{Al}$: Separation energy $= 13.059$ MeV

A neutron $\left({}_0n^1\right)$ is removed from a ${}^{41}_{20}\mathrm{Ca}$ nucleus. The corresponding nuclear reaction can be written as:

$${}^{41}_{20}\mathrm{Ca} \longrightarrow {}^{40}_{20}\mathrm{Ca} + {}^{1}_{0}\mathrm{n}$$

It is given that:

Mass $m\left({}^{40}_{20}\mathrm{Ca}\right)$ = 39.962591 u

Mass $m\left({}^{41}_{20}\mathrm{Ca}\right)$) = 40.962278 u

Mass $m\left({}_0n^1\right)$ = 1.008665 u

The mass defect of this reaction is given as:

Δm = $m\left({}^{40}_{20}\mathrm{Ca}\right) + \left({}^{1}_{0}\mathrm{n}\right) - m\left({}^{41}_{20}\mathrm{Ca}\right)$

$$= 39.962591 + 1.008665 - 40.962278 = 0.008978 \text{ u}$$

$$\text{But } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$\therefore \Delta m = 0.008978 \times 931.5 \text{ MeV}/c^2$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.008978 \times 931.5 = 8.363007 \text{ MeV}$$

For ${}^{27}_{13}\text{Al}$, the neutron removal reaction can be written as:

$${}^{27}_{13}\text{Al} \longrightarrow {}^{26}_{13}\text{Al} + {}^{1}_{0}\text{n}$$

It is given that:

Mass $m\left({}^{27}_{13}\text{Al}\right)$ = 26.981541 u

Mass $m\left({}^{26}_{13}\text{Al}\right)$ = 25.986895 u

The mass defect of this reaction is given as:

$$\Delta m = m\left({}^{26}_{13}\text{Al}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{27}_{13}\text{Al}\right)$$

$$= 25.986895 + 1.008665 - 26.981541$$
$$= 0.014019 \text{ u}$$
$$= 0.014019 \times 931.5 \text{ MeV}/c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.014019 \times 931.5 = 13.059 \text{ MeV}$$

Question 13.25:

A source contains two phosphorous radio nuclides ${}^{32}_{15}\text{P}$ ($T_{1/2}$ = 14.3d) and ${}^{33}_{15}\text{P}$ ($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from ${}^{33}_{15}\text{P}$. How long one must wait until 90% do so?

Answer

Half life of $^{32}_{15}\mathrm{P}$, $T_{1/2}$ = 14.3 days

Half life of $^{33}_{15}\mathrm{P}$, $T'_{1/2}$ = 25.3 days

$^{33}_{15}\mathrm{P}$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of $^{33}_{15}\mathrm{P}$ nucleus and 90% of $^{32}_{15}\mathrm{P}$ nucleus.

Suppose after *t* days, the source has 10% of $^{32}_{15}\mathrm{P}$ nucleus and 90% of $^{33}_{15}\mathrm{P}$ nucleus.

<u>Initially:</u>

Number of $^{33}_{15}\mathrm{P}$ nucleus = *N*

Number of $^{32}_{15}\mathrm{P}$ nucleus = 9 *N*

<u>Finally:</u>

Number of $^{33}_{15}\mathrm{P}\ \text{nucleus} = 9\ \mathrm{N'}$

Number of $^{32}_{15}\mathrm{P}\ \text{nucleus} = \mathrm{N'}$

For $^{32}_{15}\mathrm{P}$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N' = 9N(2)^{\frac{-t}{14.3}} \qquad \dots (1)$$

For $^{33}_{15}\mathrm{P}$, we can write the number ratio as:

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T'_{1/2}}}$$

$$9N' = N(2)^{\frac{-t}{25.3}} \qquad \dots (2)$$

On dividing equation (1) by equation (2), we get:

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\frac{1}{81} = 2^{-\left(\frac{11t}{25.3 \times 14.3}\right)}$$

$$\log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3} \log 2$$

$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5 \text{ days}$$

Hence, it will take about 208.5 days for 90% decay of ${}_{15}P^{33}$.

Question 13.26:

Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Answer

Take a ${}^{14}_{6}\text{C}$ emission nuclear reaction:

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

We know that:

Mass of ${}^{223}_{88}\text{Ra}$, m_1 = 223.01850 u

Mass of ${}^{209}_{82}\text{Pb}$, m_2 = 208.98107 u

Mass of ${}^{14}_{6}\text{C}$, m_3 = 14.00324 u

Hence, the Q-value of the reaction is given as:

$Q = (m_1 - m_2 - m_3)\, c^2$

= (223.01850 − 208.98107 − 14.00324) c^2

= (0.03419 c^2) u

But 1 u = 931.5 MeV/c^2

∴Q = 0.03419 × 931.5

= 31.848 MeV

Hence, the Q-value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a ${}^{4}_{2}\text{He}$ emission nuclear reaction:

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{229}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

We know that:

Mass of ${}^{223}_{88}\text{Ra}$, m_1 = 223.01850

Mass of ${}^{219}_{82}\text{Rn}$, m_2 = 219.00948

Mass of ${}^{4}_{2}\text{He}$, m_3 = 4.00260

Q-value of this nuclear reaction is given as:

$Q = (m_1 - m_2 - m_3)\, c^2$

= (223.01850 − 219.00948 − 4.00260) C^2

= (0.00642 c^2) u

= 0.00642 × 931.5 = 5.98 MeV

Hence, the Q value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically allowed.

Question 13.27:

Consider the fission of ${}^{238}_{92}\text{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are

$^{140}_{58}\mathrm{Ce}$ and $^{99}_{44}\mathrm{Ru}$. Calculate Q for this fission process. The relevant atomic and particle masses are

$m\left(^{238}_{92}\mathrm{U}\right)$ =238.05079 u

$m\left(^{140}_{58}\mathrm{Ce}\right)$ =139.90543 u

$m\left(^{99}_{44}\mathrm{Ru}\right)$ = 98.90594 u

Answer

In the fission of $^{238}_{92}\mathrm{U}$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as:

$$^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{140}_{58}\mathrm{Ce} + {}^{99}_{44}\mathrm{Ru} + 10\,{}^{0}_{-1}e$$

It is given that:

Mass of a nucleus $^{238}_{92}\mathrm{U}$, m_1 = 238.05079 u

Mass of a nucleus $^{140}_{58}\mathrm{Ce}$, m_2 = 139.90543 u

Mass of a nucleus $^{99}_{44}\mathrm{Ru}$, m_3 = 98.90594 u

Mass of a neutron $^{1}_{0}\mathrm{n}$, m_4 = 1.008665 u

Q-value of the above equation,

$$Q = \left[m'\left(^{238}_{92}\mathrm{U}\right) + m\left(^{1}_{0}\mathrm{n}\right) - m'\left(^{140}_{58}\mathrm{Ce}\right) - m'\left(^{99}_{44}\mathrm{Ru}\right) - 10m_e\right]c^2$$

Where,

m' = Represents the corresponding atomic masses of the nuclei

$m'\left(^{238}_{92}\mathrm{U}\right) = m_1 - 92m_e$

$m'\left(^{140}_{58}\mathrm{Ce}\right) = m_2 - 58m_e$

$m'\left(^{99}_{44}\mathrm{Ru}\right) = m_3 - 44m_e$

$m\left(^{1}_{0}\mathrm{n}\right) = m_4$

$$Q = [m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e]c^2$$
$$= [m_1 + m_4 - m_2 - m_3]c^2$$
$$= [238.0507 + 1.008665 - 139.90543 - 98.90594]c^2$$
$$= [0.247995\ c^2]\ \text{u}$$

But $1\ \text{u} = 931.5\ \text{MeV}/c^2$

$\therefore Q = 0.247995 \times 931.5 = 231.007\ \text{MeV}$

Hence, the *Q*-value of the fission process is 231.007 MeV.

Question 13.28:

Consider the D−T reaction (deuterium−tritium fusion)

$${}_1^2\text{H} + {}_1^3\text{H} \longrightarrow {}_2^4\text{He} + \text{n}$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m({}_1^2\text{H})$ = 2.014102 u

$m({}_1^3\text{H})$ = 3.016049 u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = 2(3*kT*/2); *k* = Boltzman's constant, *T* = absolute temperature.)

Answer

(a) Take the D-T nuclear reaction: ${}_1^2\text{H} + {}_1^3\text{H} \longrightarrow {}_2^4\text{He} + \text{n}$

It is given that:

Mass of ${}_1^2\text{H}$, m_1= 2.014102 u

Mass of ${}_1^3\text{H}$, m_2 = 3.016049 u

Mass of ${}_2^4\text{He}$ m_3 = 4.002603 u

Mass of ${}_0^1\text{n}$, m_4 = 1.008665 u

Q-value of the given D-T reaction is:

$Q = [m_1 + m_2 - m_3 - m_4]\ c^2$

$= [2.014102 + 3.016049 - 4.002603 - 1.008665]\ c^2$

$= [0.018883\ c^2]$ u

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.018883 \times 931.5 = 17.59$ MeV

(b) Radius of deuterium and tritium, $r \approx 2.0$ fm $= 2 \times 10^{-15}$ m

Distance between the two nuclei at the moment when they touch each other, $d = r + r = 4 \times 10^{-15}$ m

Charge on the deuterium nucleus = e

Charge on the tritium nucleus = e

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi \epsilon_0 (d)}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14}\ \text{J}$$

$$= \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} = 3.6 \times 10^5\ \text{eV} = 360\ \text{keV}$$

Hence, 5.76×10^{-14} J or 360 keV of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.

However, it is given that:

$$KE = 2 \times \frac{3}{2}kT$$

Where,

k = Boltzmann constant = 1.38×10^{-23} m^2 kg s^{-2} K^{-1}

T = Temperature required for triggering the reaction

$$\therefore T = \frac{KE}{3K}$$

$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^{9} \text{ K}$$

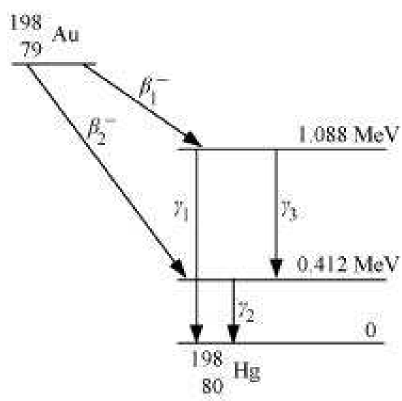
Hence, the gas must be heated to a temperature of 1.39×10^9 K to initiate the reaction.

Question 13.29:

Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

m (^{198}Au) = 197.968233 u

m (^{198}Hg) =197.966760 u



Answer

It can be observed from the given γ-decay diagram that γ_1 decays from the 1.088 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_1-decay is given as:

E_1 = 1.088 − 0 = 1.088 MeV

$h\nu_1$= $1.088 \times 1.6 \times 10^{-19} \times 10^6$ J

Where,

h = Planck's constant = 6.6×10^{-34} Js

v_1 = Frequency of radiation radiated by γ_1-decay

$$\therefore v_1 = \frac{E_1}{h}$$

$$= \frac{1.088 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 2.637 \times 10^{20} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_2-decay is given as:

E_2 = 0.412 − 0 = 0.412 MeV

$hv_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6$ J

Where,

v_2 = Frequency of radiation radiated by γ_2-decay

$$\therefore v_2 = \frac{E_2}{h}$$

$$= \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 9.988 \times 10^{19} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_3 decays from the 1.088 MeV energy level to the 0.412 MeV energy level.

Hence, the energy corresponding to γ_3-decay is given as:

E_3 = 1.088 − 0.412 = 0.676 MeV

$hv_3 = 0.676 \times 10^{-19} \times 10^6$ J

Where,

v_3 = Frequency of radiation radiated by γ_3-decay

$$\therefore v_3 = \frac{E_3}{h}$$

$$= \frac{0.676 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 1.639 \times 10^{20} \text{ Hz}$$

Mass of $m\left({}^{198}_{78}\text{Au}\right)$ = 197.968233 u

Mass of $m\left({}^{198}_{80}\text{Hg}\right)$ = 197.966760 u

1 u = 931.5 MeV/c^2

Energy of the highest level is given as:

$$E = \left[m\left({}^{198}_{78}\mathrm{Au}\right) - m\left({}^{190}_{80}\mathrm{Hg}\right)\right]$$

$$= 197.968233 - 197.966760 = 0.001473 \text{ u}$$
$$= 0.001473 \times 931.5 = 1.3720995 \text{ MeV}$$

β_1 decays from the 1.3720995 MeV level to the 1.088 MeV level

∴Maximum kinetic energy of the β_1 particle = 1.3720995 − 1.088

= 0.2840995 MeV

β_2 decays from the 1.3720995 MeV level to the 0.412 MeV level

∴Maximum kinetic energy of the β_2 particle = 1.3720995 − 0.412

= 0.9600995 MeV

Question 13.30:

Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Answer

(a) Amount of hydrogen, *m* = 1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen (${}^{1}_{1}\mathrm{H}$) contains 6.023 × 10^{23} atoms.

∴1000 g of ${}^{1}_{1}\mathrm{H}$ contains 6.023 × 10^{23} × 1000 atoms.

Within the sun, four ${}_{1}^{1}\text{H}$ nuclei combine and form one ${}_{2}^{4}\text{He}$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg ${}_{1}^{1}\text{H}$ is:

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^3}{4}$$
$$= 39.1495 \times 10^{26} \text{ MeV}$$

(b) Amount of ${}_{92}^{235}\text{U}$ = 1 kg = 1000 g

1 mole, i.e., 235 g of ${}_{92}^{235}\text{U}$ contains 6.023×10^{23} atoms.

$\therefore$1000 g of ${}_{92}^{235}\text{U}$ contains $\dfrac{6.023 \times 10^{23} \times 1000}{235}$ atoms

It is known that the amount of energy released in the fission of one atom of ${}_{92}^{235}\text{U}$ is 200 MeV.

Hence, energy released from the fission of 1 kg of ${}_{92}^{235}\text{U}$ is:

$$E_2 = \frac{6 \times 10^{23} \times 1000 \times 200}{235}$$
$$= 5.106 \times 10^{26} \text{ MeV}$$

$$\therefore \frac{E_1}{E_1} = \frac{39.1495 \times 10^{26}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

Question 13.31:

Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Answer

Amount of electric power to be generated, $P = 2 \times 10^5$ MW

10% of this amount has to be obtained from nuclear power plants.

∴Amount of nuclear power, $P_1 = \frac{10}{100} \times 2 \times 10^5$

$= 2 \times 10^4$ MW

$= 2 \times 10^4 \times 10^6$ J/s

$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365$ J/y

Heat energy released per fission of a ^{235}U nucleus, E = 200 MeV

Efficiency of a reactor = 25%

Hence, the amount of energy converted into the electrical energy per fission is calculated as:

$$\frac{25}{100} \times 200 = 50 \text{ MeV}$$

$$= 50 \times 1.6 \times 10^{-19} \times 10^6 = 8 \times 10^{-12} \text{ J}$$

Number of atoms required for fission per year:

$$\frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24} \text{ atoms}$$

1 mole, i.e., 235 g of U^{235} contains 6.023×10^{23} atoms.

$\therefore$Mass of 6.023×10^{23} atoms of U^{235} = 235 g = 235×10^{-3} kg

$\therefore$Mass of 78840×10^{24} atoms of U^{235}

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24}$$

$$= 3.076 \times 10^{4} \text{ kg}$$

Hence, the mass of uranium needed per year is 3.076×10^{4} kg.

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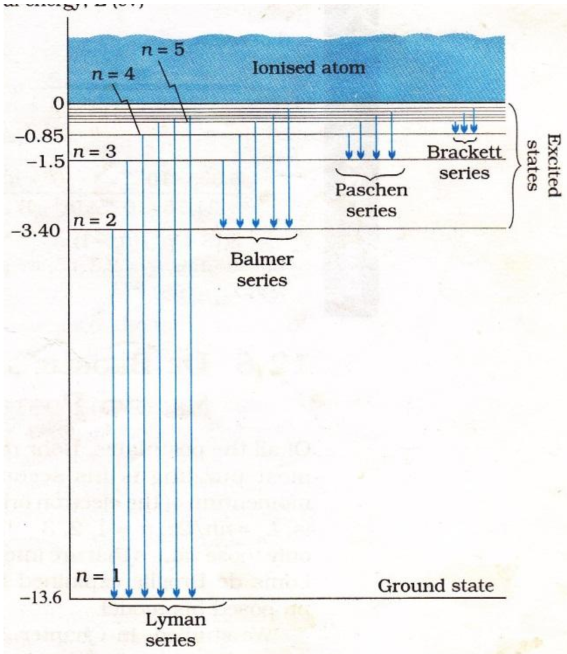
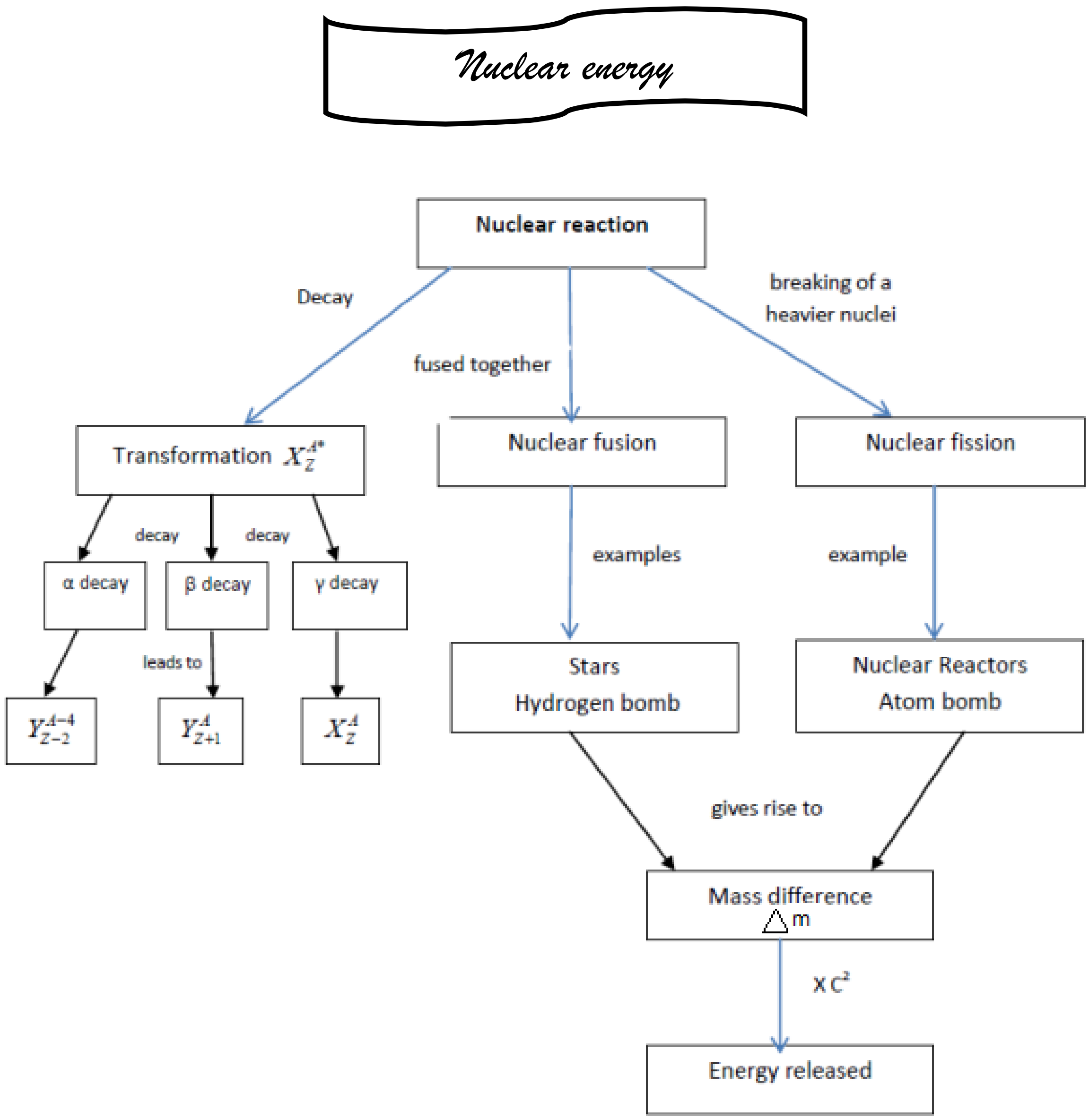
8. ATOMS & NUCLEI

GIST	
Thomson's model of atom- Every atom consists of fuels charged sphere in which electrons are embedded like seeds in water melon.	Its drawbacks: couldn't explain large angle scattering & the origin of spectral series.
Rutherford's model of atom- i) Every atom consists of a tiny central core, called the atomic nucleus, in which the entire positive charge and almost entire mass of the atom are concentrated. ii) The size of nucleus is of the order of 10^{-15}m , which is very small as compared to the size of the atom which is of the order of 10^{-10}m. iii)The atomic nucleus is surrounded by certain number of electrons. As atom on the whole is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus. iv)These electrons revolve around the nucleus in various circular orbits as do the planets around the aun. The centripetal force required by electron for revolution is provided by the electrostatic force of attraction between the electrons and the nucleus.	Limitations: couldn't explain the stability of the nucleus & the emission of line spectra of fixed frequencies.
Distance of closest approach of the alpha particle in the α particle scattering experiment	$r_0=\frac{2kZe^2}{1/2mv^2}$
Impact parameter of the alpha particle	$b=\frac{kZe^2\cot\theta/2}{1/2mv^2}$
Bohr's model of atom	Limitations-applicable only for hydrogen like atoms & couldn't explain the splitting of spectral lines. (not consider electro static force among the electrons)
Orbit radius of the electron around the nucleus	$r=e^2/4\pi\epsilon_0 mv^2$, $v=2\pi ke^2 / nh$, $r=n^2h^2mke^2$
Energy of the electron in the nth orbit of hydrogen atom	$E_n= -me^4/8\epsilon_0^2n^2h^2 = -13.6/n^2$ eV $E=-2.18*10^{-18}$ J / n^2
• Angular momentum of electron in any orbit is	$L = mvr = nh/2\pi$, n=1,2,3,…

integral multiple of h/2π	
• Wave number $\overline{v}$ 	$1/\lambda = R(1/n_1^2 - 1/n_2^2)$ $R=1.097 * 10^{+7} m^{-1}$
Atomic Number (Z)	No of protons in a nucleus
Mass Number (A) Number of neutrons	No. of nucleons(protons + neutrons) in a nucleus A-Z
Nuclear radius	$R=R_0 A^{1/3}$
Nuclear density	$P= 3m/4\pi R_0^3$
Isotopes	Same Z & different A Ex, $^1H_2$,$^1H_3$,$^1h_1$, & C^{12},C^{14},C^{16}
Isobars	Same A & different Z [$_{18}Ar^{40}$,$_{20}Co^{40}$] & ($_1H^3$, $_2H^3$)
Isotones Map defect △ m	Same no. of neutrons Mass of neutrons – $_1H^3$, $_2He^4$
Binding energy E_b	E= △ m x c^2 (△ m= mass of reactants – mass of products) 1 a.m.u.= 931.5 Mev
Radioactive decay law	$dN/dt=-\lambda N$ -dW/dt= R= Activity unit Bq.
No: of nuclei remaining un-decayed at any instant of time	$N =N_0 e^{-\lambda t}$ OR $N=N_0(½)^n$, $n = t/t_{1/2}$
Half life	$t_{1/2}=\frac{0.693}{\lambda}$
Mean life	$\tau= 1/\lambda$
3 types of radiations	Alpha,beta,gamma

Nuclear fission	Splitting of a heavy nucleus into lighter elements.This process is made use of in Nuclear reactor & Atom bomb Nuclear Reactor is based upon controlled nuclear chain reaction and has 1) Nuclear fuel 2) modulator 3) control rods 4) coolant 5) shielding
Nuclear fusion	Fusing of lighter nuclei to form a heavy nucleus.This process takes place in Stars & Hydrogen bomb. Controlled Thermonuclear Fusion In a fusion reactor- a) high particle density is required b) high plasma temperature of 10^9K c) a long confinement time is required

CONCEPT MAP


Nuclear energy
Nuclear reaction
Decay
fused together
breaking of a heavier nuclei
Transformation X_Z^{A*}
Nuclear fusion
Nuclear fission
decay
decay
α decay
β decay
γ decay
leads to
Y_{Z-2}^{A-4}
Y_{Z+1}^{A}
X_Z^A
examples
example
Stars
Hydrogen bomb
Nuclear Reactors
Atom bomb
gives rise to
Mass difference
△m
X C^2
Energy released

QUESTIONS

ALPHA PARTICLE SCATTERING

1. What is the distance of closest approach when a 5Mev proton approaches a gold nucleus (Z=79) (1)

Ans $r_{0=} \frac{1}{4\pi\varepsilon_\circ} \frac{Ze^2}{F_2}$ = 2.3 * 10^{-14}m.

2. Which has greater ionizing power: alpha or beta particle? (1)

BOHR'S ATOMIC MODEL

1. In Bohr's theory of model of a Hydrogen atom, name the physical quantity which equals to an integral multiple of h/2∏? (1)

***Ans**: Angular momentum*

2. What is the relation between 'n' & radius 'r' of the orbit of electron in a Hydrogen atom according to Bohr's theory? (1)

***Ans**: $r \; \alpha \; n^2$*

3. What is Bohr's quantization condition? (1)

*4. For an electron in the second orbit of hydrogen, what is the moment of linear momentum as per the Bohr's model? (2)

***Ans**: L=2(h/2∏) =h/∏ (moment of linear momentum is angular momentum)*

5. Calculate the ratio of energies of photons produced due to transition of electron of hydrogen atoms from 2^{nd} level to 1^{st} and highest level to second level. (3)

$E_{2\text{-}1}$ = Rhc[$1/n_1^2 - 1/n^2$] = ¾ Rhc

E∞ - E1 = Rhc($1/2^2$ – 1/∞) = Rhc / 4

SPECTRAL SERIES

*1. What is the shortest wavelength present in the Paschen series of hydrogen spectrum? (2)

***Ans**: n_1=3, n_2=infinity, λ=9/R=8204Å*

2. Calculate the frequency of the photon which can excite an electron to -3.4 eV from -13.6 eV.

***Ans**: 2.5x10^{15}Hz* (2)

3. The wavelength of the first member of Balmer series in the hydrogen spectrum is 6563Å.Calculate the wavelength of the first member of Lyman series in the same spectrum.

***Ans**: 1215.4Å* (2)

4. The ground state energy of hydrogen atom is -13.6eV.What is the K.E & P.E of the electron in this state? (2)

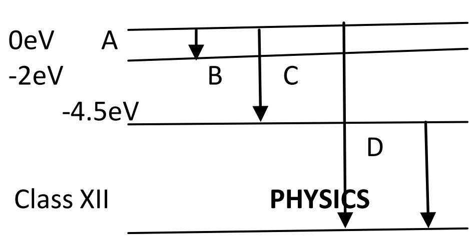
***Ans**: K.E=-E=13.6 eV, P.E=-2K.E=-27.2 eV*

*5. Find the ratio of maximum wavelength of Lyman series in hydrogen spectrum to the maximum wavelength in Paschen Series? (2)

***Ans**: 7:108*

*6. The energy levels of an atom are as shown below. a) Which of them will result in the transition of a photon of wavelength 275 nm? b) Which transition corresponds to the emission of radiation maximum wavelength?

(3)



-10eV

***Ans**: E=hc/λ=4.5eV, transition B   Eα1/λ, transition A*

*7. The spectrum of a star in the visible & the ultraviolet region was observed and the wavelength of some of the lines that could be identified were found to be 824Å,970Å,1120Å,2504Å,5173Å &6100Å.Which of these lines cannot belong to hydrogen spectrum? (3)

***Ans**: 970Å*

(3)

9. What is the energy possessed by an ē for n=∞ ?

Ans E=0 (1)

10. Calculate the ratio of wavelength of photon emitted due to transition of electrons of hydrogen atom from

i) Second permitted level to first level

ii) Highest permitted level to second level (3)

11. The radius of inner most electron orbit of H_2 atom is 5.3 x 10^{-11}m. What are radii for n=2, 3, 4? (3)

Ans: $r_n = n^2 r_1$

COMPOSITION OF NUCLEUS

1. What is the relation between the radius of the atom & the mass number? (1)

***Ans**: size α $A^{1/3}$*

2. What is the ratio of the nuclear densities of two nuclei having mass numbers in the ratio 1:4?

***Ans**: 1:1* (1)

3. How many electrons, protons & neutrons are there in an element of atomic number (Z) 11& mass number (A) 24? (1)

Hint: $n_e = n_p = 11$, $n_n = (A – Z) = 24 - 11 = 13$

4. Select the pairs of isotopes & isotones from the following: (2)

i. $^{13}C_6$ ii. $^{14}N_7$ iii. $^{30}P_{15}$ iv. $^{31}P_{15}$

***Ans**: isotopes-iii &iv ,isotones-i& ii*

5. By what factor must the mass number change for the nuclear radius to become twice? (2)

$\sqrt[3]{2}\ or\ 2^{\frac{1}{3}}\ time A$

NUCLEAR FORCE & BINDING ENERGY.

1. What is the nuclear force? Mention any two important properties of it.

(2)

2. Obtain the binding energy of the nuclei $^{56}Fe_{26}$ & $^{209}Bi_{83}$ in MeV from the following data: m_H=1.007825amu, m_n=1.008665amu, m($^{56}Fe_{26}$)=55.934939amu, m($^{209}Bi_{83}$)=208.980388amu, 1amu=931.5MeV

3. Which nucleus has the highest binding energy per nucleon? (3)

***Ans**: Fe →492.26MeV,8.79MeV/A Bi →1640.3MeV,7.85 MeV*

Hence $^{56}Fe_{26}$

4. From the given data, write the nuclear reaction for α decay of $^{238}_{92}U$ and hence calculate the energy released. $^{238}_{92}U$ = 238.050794u $^{4}_{2}He$ = 4.00260u $^{234}_{90}Th$ = 234.04363u (3)

5Binding Energy of $_8O^{16}$ & $_{17}C^{35}$ one 127.35 Mev and 289.3 Mev respectively. Which of the two nuclei is more stable stability & BE/N? (2)

RADIOACTIVITY

1. How is a β particle different from an electron? (1)

2. Draw graph between no. of nuclei un-decayed with time for a radioactive substance (1)

3. Among the alpha, beta & gamma radiations, which are the one affected by a magnetic field? (1)

***Ans**: alpha & beta*

4. Why do α particles have high ionizing power? (1)

***Ans**: because of their large mass & large nuclear cross section*

5. Write the relationship between the half life & the average life of a radioactive substance. (1)

***Ans**: $T = 1.44t_{1/2}$*

6. If 70% of a given radioactive sample is left un-decayed after 20 days, what is the % of original sample will get decayed in 60 days? (2)

7. How does the neutron to proton ratio affected during (i)β decay ii) α decay (2)

8. A radioactive sample having N nuclei has activity R. Write an expression for its half life in terms of R & N. (2)

***Ans**: $R=N\lambda$, $t_{1/2}=0.693/\lambda = 0.693N/R$*

9. Tritium has a half life of 12.5 years against beta decay. What fraction of a sample of pure tritium will remain un-decayed after 25 years? (2)

***Ans**: $N_0/4$*

10. What percentage of a given mass of a radioactive substance will be left un-decayed after 5 half-life periods? (2)

***Ans**: $N/N_0 = 1/2^n = 1/32 = 3.125\%$*

11. A radioactive nucleus 'A' decays as given below:

$$A \xrightarrow{\beta} A_1 \xrightarrow{\gamma} A_2$$

If the mass number & atomic number of A_1 are 180 & 73 respectively, find the mass number & atomic number of A & A_2 (2)

***Ans**: A—180 & 72, A2—176 & 71*

12. Two nuclei P & Q have equal no: of atoms at t=0.Their half lives are 3 & 9 hours respectively. Compare the rates of disintegration after 18 hours from the start. (2)

***Ans**: 3:16*

*13. Two radioactive materials X_1& X_2 have decay constants 10λ & λ respectively. If initially they have the same no: of nuclei, find the time after which the ratio of the nuclei of X_1 to that of X_2 will be 1/e?

Ans: $N=N_0e^{-\lambda t}$, $t=1/9\lambda$ (3)

*14. One gram of radium is reduced by 2.1mg in 5 years by decay. Calculate the half-life of Uranium.

***Ans:** 1672 years* (3)

*16. At a given instant there are 25% un-decayed radioactive nuclei in a sample. After 10 seconds the number of un-decayed nuclei reduces to 12.5 %.calculate the i) mean life of the nuclei ii) the time in which the number of the un-decayed nuclei will further reduce to 6.25 % of the reduced number.

***Ans**: $t_{1/2}=10s$, $\lambda=.0693/s$, $\tau=1/\lambda=14.43s$, $N=1/16(N_0/8) \rightarrow t=4x10=40s$* (3)

17. Half lives of two substances A and B are 20 min and 40 min respectively. Initially the sample had equal no of nuclei. Find the ratio of the remaining no: of nuclei of A and B after 80 min.

Ans: 1:4 (3)

<u>NUCLEAR REACTIONS</u>

1. Why heavy water is often used in a nuclear reactor as a moderator? (1)

2. Why is neutron very effective as a bombarding particle in a nuclear reaction? (1)

***Ans**: Being neutral it won't experience any electrostatic force of attraction or repulsion.*

3. Why is the control rods made of cadmium? (1)

***Ans**: They have a very high affinity on neutrons.*

4. Name the phenomenon by which the energy is produced in stars. (1)

***Ans**: Uncontrolled Nuclear fusion*

5. Name the physical quantities that remain conserved in a nuclear reaction? (1)

6. What is neutron multiplication factor? For what value of this, a nuclear reactor is said to be critical?

Ans: K=1 (2)

7. 4 nuclei of an element fuse together to form a heavier nucleus .If the process is accompanied by release of energy, which of the two: the parent or the daughter nuclei would have higher binding energy per nucleon. Justify your answer. (2)

8. If 200MeV energy is released in the fission of single nucleus of ${}^{235}_{92}U$, how much fission must occur to produce a power of 1 kW. (3)