Exercise 12.1

Question 1:

A point is on the *x*-axis. What are its *y*-coordinates and *z*-coordinates?

Answer

If a point is on the *x*-axis, then its *y*-coordinates and *z*-coordinates are zero.

Question 2:

A point is in the XZ-plane. What can you say about its *y*-coordinate?

Answer

If a point is in the XZ plane, then its *y*-coordinate is zero.

Question 3:

Name the octants in which the following points lie:

(1, 2, 3), (4, −2, 3), (4, −2, −5), (4, 2, −5), (−4, 2, −5), (−4, 2, 5),
(−3, −1, 6), (2, −4, −7)

Answer

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant **I**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, −2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, −2, −5) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, 2, −5) are positive, positive, and negative respectively. Therefore, this point lies in octant **V**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (−4, 2, −5) are negative, positive, and negative respectively. Therefore, this point lies in octant **VI**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (−4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant **II**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (−3, −1, 6) are negative, negative, and positive respectively. Therefore, this point lies in octant **III**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (2, −4, −7) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

Question 4:

Fill in the blanks:

Answer

(i) The *x*-axis and *y*-axis taken together determine a plane known as $\underline{\text{XY} - \text{plane}}$.

(ii) The coordinates of points in the XY-plane are of the form $\underline{(x, y, 0)}$.

(iii) Coordinate planes divide the space into $\underline{\text{eight}}$ octants.

Exercise 12.2

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4) (iv) (2, –1, 3) and (–2, 1, 3)

Answer

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$
$$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

(ii) Distance between points (–3, 7, 2) and (2, 4, –1)

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

(iii) Distance between points (–1, 3, –4) and (1, –3, 4)

$$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$$
$$= \sqrt{(2)^2 + (-6)^3 + (8)^2}$$
$$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$$

(iv) Distance between points (2, –1, 3) and (–2, 1, 3)

$$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$$
$$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

Question 2:

Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Answer

Let points (–2, 3, 5), (1, 2, 3), and (7, 0, –1) be denoted by P, Q, and R respectively.

Points P, Q, and R are collinear if they lie on a line.

$$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$$
$$= \sqrt{(3)^2 + (-1)^2 + (-2)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$

$$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

Here, PQ + QR $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = PR

Hence, points P(−2, 3, 5), Q(1, 2, 3), and R(7, 0, −1) are collinear.

Question 3:

Verify the following:

(i) (0, 7, −10), (1, 6, −6) and (4, 9, −6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) are the vertices of a right angled triangle.

(iii) (−1, 2, 1), (1, −2, 5), (4, −7, 8) and (2, −3, 4) are the vertices of a parallelogram.

Answer

(i) Let points (0, 7, −10), (1, 6, −6), and (4, 9, −6) be denoted by A, B, and C respectively.

$$\begin{aligned} AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\ &= \sqrt{(3)^2 + (3)^2} \\ &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} CA &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\ &= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{16+4+16} = \sqrt{36} = 6 \end{aligned}$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let (0, 7, 10), (−1, 6, 6), and (−4, 9, 6) be denoted by A, B, and C respectively.

$$\begin{aligned}AB &= \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}\\ &= \sqrt{(-1)^2+(-1)^2+(-4)^2}\\ &= \sqrt{1+1+16} = \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}\\ &= \sqrt{(-3)^2+(3)^2+(0)^2}\\ &= \sqrt{9+9} = \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(0+4)^2+(7-9)^2+(10-6)^2}\\ &= \sqrt{(4)^2+(-2)^2+(4)^2}\\ &= \sqrt{16+4+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

Now, $AB^2+BC^2=\left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2=18+18=36=AC^2$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) be denoted by A, B, C, and D respectively.

$$\text{AB} = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$\text{BC} = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

$$\text{CD} = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$\text{DA} = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Answer

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, –1).

Accordingly, PA = PB

$$\Rightarrow \text{PA}^2 = \text{PB}^2$$
$$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question 5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (–4, 0, 0) is equal to 10.

Answer

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (–4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$$

$$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$$

$$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$$

$$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

Exercise 12.3

Question 1:

Find the coordinates of the point which divides the line segment joining the points (–2, 3, 5) and (1, –4, 6) in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Answer

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right).$$

Let R (x, y, z) be the point that divides the line segment joining points(–2, 3, 5) and (1, –4, 6) internally in the ratio 2:3

$$x=\frac{2(1)+3(-2)}{2+3}, y=\frac{2(-4)+3(3)}{2+3}, \text{ and } z=\frac{2(6)+3(5)}{2+3}$$

$$\text{i.e., } x=\frac{-4}{5}, y=\frac{1}{5}, \text{ and } z=\frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5},\frac{1}{5},\frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio m: n are

$$\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n},\frac{mz_2-nz_1}{m-n}\right).$$

Let R (x, y, z) be the point that divides the line segment joining points(–2, 3, 5) and (1, –4, 6) externally in the ratio 2:3

$$x=\frac{2(1)-3(-2)}{2-3}, y=\frac{2(-4)-3(3)}{2-3}, \text{ and } z=\frac{2(6)-3(5)}{2-3}$$

$$\text{i.e., } x=-8, y=17, \text{ and } z=3$$

Thus, the coordinates of the required point are (–8, 17, 3).

Question 2:

Given that P (3, 2, −4), Q (5, 4, −6) and R (9, 8, −10) are collinear. Find the ratio in which Q divides PR.

Answer

Let point Q (5, 4, −6) divide the line segment joining points P (3, 2, −4) and R (9, 8, −10) in the ratio *k*:1.

Therefore, by section formula,

$$(5,4,-6)=\left(\frac{k(9)+3}{k+1},\frac{k(8)+2}{k+1},\frac{k(-10)-4}{k+1}\right)$$

$$\Rightarrow \frac{9k+3}{k+1}=5$$
$$\Rightarrow 9k+3=5k+5$$
$$\Rightarrow 4k=2$$
$$\Rightarrow k=\frac{2}{4}=\frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points (−2, 4, 7) and (3, −5, 8).

Answer

Let the YZ planedivide the line segment joining points (−2, 4, 7) and (3, −5, 8) in the ratio *k*:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1},\frac{k(-5)+4}{k+1},\frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the *x*-coordinate of any point is zero.

$$\frac{3k-2}{k+1}=0$$
$$\Rightarrow 3k-2=0$$
$$\Rightarrow k=\frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Question 4:

Using section formula, show that the points A (2, −3, 4), B (−1, 2, 1) and $C\left(0,\frac{1}{3},2\right)$ are collinear.

Answer

The given points are A (2, −3, 4), B (−1, 2, 1), and $C\left(0,\frac{1}{3},2\right)$.

Let P be a point that divides AB in the ratio *k*:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1},\frac{k(2)-3}{k+1},\frac{k(1)+4}{k+1}\right)$$

Now, we find the value of *k* at which point P coincides with point C.

By taking $\frac{-k+2}{k+1}=0$, we obtain *k* = 2.

For *k* = 2, the coordinates of point P are $\left(0,\frac{1}{3},2\right)$.

i.e., $C\left(0,\frac{1}{3},2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

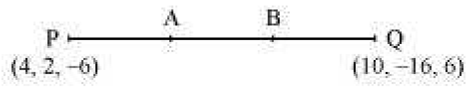
Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, −6) and Q (10, −16, 6).

Answer

Let A and B be the points that trisect the line segment joining points P (4, 2, −6) and Q (10, −16, 6)



Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2},\frac{1(-16)+2(2)}{1+2},\frac{1(6)+2(-6)}{1+2}\right)=(6,-4,-2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1},\frac{2(-16)+1(2)}{2+1},\frac{2(6)-1(6)}{2+1}\right)=(8,-10,2)$$

Thus, (6, −4, −2) and (8, −10, 2) are the points that trisect the line segment joining points P (4, 2, −6) and Q (10, −16, 6).

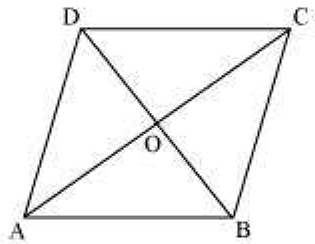
NCERT Miscellaneous Solutions

Question 1:

Three vertices of a parallelogram ABCD are A (3, −1, 2), B (1, 2, −4) andC (−1, 1, 2). Find the coordinates of the fourth vertex.

Answer

The three vertices of a parallelogram ABCD are given as A (3, −1, 2), B (1, 2, −4), and C (−1, 1, 2). Let the coordinates of the fourth vertex be D (*x*, *y*, *z*).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

∴Mid-point of AC = Mid-point of BD

$$\Rightarrow\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$

$$\Rightarrow(1,0,2)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$

$$\Rightarrow\frac{x+1}{2}=1,\ \frac{y+2}{2}=0,\ \text{and}\ \frac{z-4}{2}=2$$

⇒ $x = 1$, $y = -2$, and $z = 8$

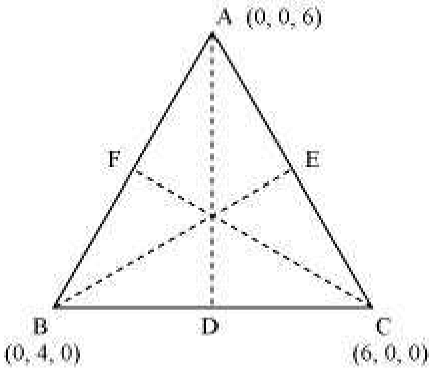
Thus, the coordinates of the fourth vertex are (1, −2, 8).

Question 2:

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Answer

Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

$\therefore$Coordinates of point D = $\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0+0}{2}\right)$ = (3, 2, 0)

$\text{AD}=\sqrt{(0-3)^2+(0-2)^2+(6-0)^2}=\sqrt{9+4+36}=\sqrt{49}=7$

Since BE is the median, E is the mid-point of AC.

$\therefore$ Coordinates of point E $=\left(\frac{0+6}{2},\frac{0+0}{2},\frac{6+0}{2}\right)=(3,0,3)$

$\text{BE}=\sqrt{(3-0)^2+(0-4)^2+(3-0)^2}=\sqrt{9+16+9}=\sqrt{34}$

Since CF is the median, F is the mid-point of AB.

$\therefore$ Coordinates of point F $=\left(\frac{0+0}{2},\frac{0+4}{2},\frac{6+0}{2}\right)=(0,2,3)$

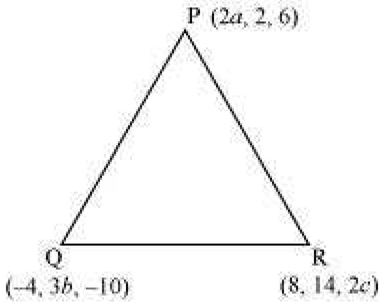
Length of CF $=\sqrt{(6-0)^2+(0-2)^2+(0-3)^2}=\sqrt{36+4+9}=\sqrt{49}=7$

Thus, the lengths of the medians of ΔABC are $7, \sqrt{34}$, and 7.

Question 3:

If the origin is the centroid of the triangle PQR with vertices P (2*a*, 2, 6), Q (–4, 3*b*, –10) and R (8, 14, 2*c*), then find the values of *a*, *b* and *c*.

Answer



It is known that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Therefore, coordinates of the centroid of ΔPQR

$$=\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of ΔPQR.

$$\therefore (0,0,0)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

$$\Rightarrow \frac{2a+4}{3}=0, \frac{3b+16}{3}=0 \text{ and } \frac{2c-4}{3}=0$$

$$\Rightarrow a=-2, b=-\frac{16}{3} \text{ and } c=2$$

Thus, the respective values of *a*, *b*, and *c* are $-2, -\frac{16}{3}$, and 2.

Question 4:

Find the coordinates of a point on *y*-axis which are at a distance of $5\sqrt{2}$ from the point P (3, −2, 5).

Answer

If a point is on the *y*-axis, then *x*-coordinate and the *z*-coordinate of the point are zero.

Let A (0, *b*, 0) be the point on the *y*-axis at a distance of $5\sqrt{2}$ from point P (3, −2, 5).

Accordingly, $AP = 5\sqrt{2}$

$\therefore AP^2 = 50$

$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$

$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$

$\Rightarrow b^2 + 4b - 12 = 0$

$\Rightarrow b^2 + 6b - 2b - 12 = 0$

$\Rightarrow (b+6)(b-2) = 0$

$\Rightarrow b = -6 \text{ or } 2$

Thus, the coordinates of the required points are (0, 2, 0) and (0, –6, 0).

Question 5:

A point R with *x*-coordinate 4 lies on the line segment joining the pointsP (2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[**Hint** suppose R divides PQ in the ratio *k*: 1. The coordinates of the point R are given by

$\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$]

Answer

The coordinates of points P and Q are given as P (2, –3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio *k*:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$$

It is given that the *x*-coordinate of point R is 4.

$\therefore \frac{8k+2}{k+1} = 4$

$\Rightarrow 8k + 2 = 4k + 4$

$\Rightarrow 4k = 2$

$\Rightarrow k = \frac{1}{2}$

Therefore, the coordinates of point R are $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right) = (4, -2, 6)$

Question 6:

If A and B be the points (3, 4, 5) and (−1, 3, −7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Answer

The coordinates of points A and B are given as (3, 4, 5) and (−1, 3, −7) respectively.

Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$\begin{aligned}
PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\
&= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\
&= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50 \\
PB^2 &= (x+1)^2 + (y-3)^2 + (z+7)^2 \\
&= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59
\end{aligned}$$

Now, if $PA^2 + PB^2 = k^2$, then

$$\left(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50\right) + \left(x^2 + 2x + y^2 - 6y + z^2 + 14z + 59\right) = k^2$$

$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$

$$\Rightarrow 2\left(x^2 + y^2 + z^2 - 2x - 7y + 2z\right) = k^2 - 109$$

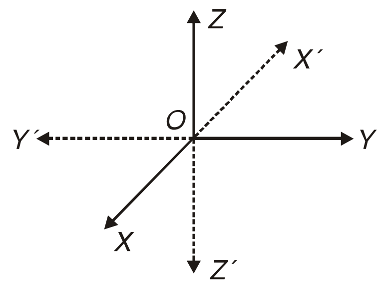
$$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \dfrac{k^2 - 109}{2}$.

CHAPTER - 12

INTRODUCTION TO THREE DIMENSIONAL COORDINATE GEOMETRY

- Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.



Coordinate axes :	XOX', YOY', ZOZ'
Coordinate planes :	XOY, YOZ, ZOX or
	XY, YX, ZX planes
Octants :	OXYZ, OX'YZ, OXY'Z, OXYZ'
	OX' Y'Z, OXY'Z', OX'YZ', OX'Y'Z'

- Coordinates of a point P are the perpendicular distances of P from three coordinate planes YZ, ZX and XY respectively.
- The distance between the point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in space and let R be a point on line segment PQ such that it divides PQ in the ratio $m_1 : m_2$

(i) internally, then the coordinates of R are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

(ii) externally, then coordinates of R are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

- Coordinates of centroid of a triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are

$$\left(\frac{x_1 + y_1 + z_1}{3}, \frac{x_2 + y_2 + z_2}{3}, \frac{x_3 + y_3 + z_3}{3}\right)$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find image of (–2, 3, 5) in YZ plane.
2. Name the octant in which (–5, 4, –3) lies.
3. Find the distance of the point P(4, –3, 5) from XY plane.
4. Find the distance of point P(3, –2,1) from z–axis.
5. Write coordinates of foot of perpendicular from (3, 7, 9) on x axis.
6. Find the distance between points (2, 3, 4) and (–1, 3, –2).
7. Find the coordinates of the foot of perpendicular drawn from the point (2, 4, 5) on y-axis.
8. Find the coordinates of foot of perpendicular from (1, –1, 1) on XY plane.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

9. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.
10. Find the point on y–axis which is equidistant from the point (3, 1, 2) and (5, 5, 2).
11. Find the coordinates of a point equidistant from four points (0,0,0), (2,0,0), (0,3,0) and (0,0,8), if it exists.

12. The centroid of $\triangle ABC$ is at (1,1,1). If coordinates of A and B are (3,–5,7) and (–1, 7, –6) respectively, find coordinates of point C.

13. Find the length of the medians of the triangle with vertices A(0, 0, 6) B(0, 4, 0) and C(6, 0, 0).

14. Find the ratio in which the line joining the points A(2, 1, 5) and B(3, 4, 3) is divided by the plane $2x + 2y - 2z = 1$. Also, find the coordinates of the point of division.

15. If the extremities (end points) of a diagonal of a square are (1,–2,3) and (2,–3,5) then find the length of the side of square.

16. Determine the point in XY plane which is equidistant from the points A (1, –1, 0) B(2, 1, 2) and C(3, 2, –1).

17. If the points A(1, 0, –6), B(–5, 9, 6) and C(–3, p, q) are collinear, find the value of p and q.

18. Show that the points A(3,3,3), B(0,6,3), C(1,7,7) and D(4,4,7) are the vertices of a square.

19. The coordinates of mid point of sides of Δ ABC are (–2, 3, 5), (4, –1, 7) and (6, 5, 3). Find the coordinates of vertices of $\triangle ABC$.

20. Find the coordinates of the point P which is five-sixth of the way from A(2, 3, –4) to B(8, –3, 2).

ANSWERS

1. (2,3,5)
2. OX' YZ'
3. 5 units
4. $\sqrt{13}$ units
5. (3,0,0)
6. $\sqrt{45}$ units
7. (0, 4, 0)
8. (1, –1, 0)
10. (0, 5, 0)
11. $\left(1, \frac{3}{2}, 4\right)$
12. (1,1,2)
13. 7, $\sqrt{34}$, 7

14. $5 : 7, \left(\frac{29}{12}, \frac{9}{4}, \frac{25}{6}\right)$

15. $\sqrt{3}$ units

16. $\left(\frac{3}{2}, 1, 0\right)$

17. p = 6, q = 2

19. $\begin{bmatrix} (0, 9, 1), \\ (-4, -3, 9), \\ (12, 1, 5) \end{bmatrix}$

20. (7,–2,1)